A Stochastic Model of Platoon Formation in Traffic Flow

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Abstract

In this paper we study formation of platoons and their velocity–size distribution in freeway traffic using the stochastic Master equation approach. The solution to the Master equation and the moments of the distribution are obtained by numeric integration. We also discuss a possible generalization of this approach to the case of a multi–lane traffic flow and propose a microscopic, particle–hopping model for simulations.

1 Introduction

In recent years there has been considerable interest in the properties of vehicular traffic flow. Experimental data from traffic observations revealed that, depending on the car density, there are several separated phases of flow: free flow, stop and go traffic, and synchronous flow. It is believed that for sufficiently high densities a hydrodynamic, coarse–grained description of traffic flow adequately describes many of the experimentally observed phenomena. For low densities, however, this continuous approximation fails, and one has to refer to computer simulations and/or different phenomenological approaches. A number of microscopic models, such as cellular automata, particle-hoping, car–following, have been proposed (for a review see Ref[1] and references therein).

One of the interesting phenomena in traffic flow is formation of clusters, or platoons, that happens even at low densities. The mechanisms for platoon formation is of a great importance for Automated Highway Systems. In AHS the vehicle control, such as speed and headway distance maintenance, is fully automatic. It has been suggested that AHS can significantly increase highway capacity by allowing cars to travel in closely packed clusters, where the intra–cluster distance can be as small as $\sim 1m$, and the headway between clusters $\sim 60m$. Such "structured" flow increases the freeway capacity, and is considered to be much safer thanks to the small variance in car velocities.

AHS can be described as an autonomous multi–agent system where each car (agent) has its own characteristics (speed,acceleration, etc.) and goals (reaching final destination in a given time, maintaining safety, etc.). Autonomous multi–agent systems that are characterized by distributed control mechanisms are much more robust compared to systems with centralized control. The absence of a centralized control requires interaction among agents to form and maintain platoons. As we mentioned above, platoons form spontaneously due to the differences in inherent velocities. If no passing is allowed, then platoons form behind the slowest cars so that the performance of the system is solely determined by the fraction of the slow cars. If passing is allowed for all agents, however, platoons might not form at all since each agent will tend to maintain its inherent velocity to minimize its travel time. In the present paper, we propose a model with "restricted" passing, where agents are allowed to pass with probability depending on the "slowdown" they will experience by joining the platoon. The higher the slowdown, the higher the probability of passing. We study this problem in the present paper by means of stochastic Master equation approach.

2 Master Equation for Platoon Formation

Application of statistical physics to the traffic problem goes back to Prigogine and Herman[2], and is based on apparent similarities of vehicular traffic flow and dilute gases. In terminology of statistical physics the state of the system is described by a distribution function $F(\vec{x}(t))$ which obeys the continuity equation analogous to the Boltzman equation from the kinetic theory of gases. The vector $\vec{x}(t)$ denotes the state of an individual vehicle, i.e., its position, velocity, lane number, etc. Below we will assume that the system is spatially homogenous, and that the state of each vehicle is described by its velocity and the size of the cluster it belongs to. For the sake of simplicity we will consider the case of one–lane traffic with passing. We will outline a simple generalization of the formalism for the case of multi–lane traffic in the last section.

Consider a segment of a freeway with on and off ramps "homogeneously" located along the freeway. Initially, isolated cars are randomly distributed on the freeway according to some density ρ_0. Each car moves with its inherent velocity drawn from a given (continuous) distribution $P_0(v)$ (we will measure the velocity, distance and time in units of the initial average velocity $v_0 = \int_0^\infty dvvP_0(v)$, inverse density, and the average collision time $1/v_0\rho_0$, respectively). When a fast car (cluster) is blocked in front by a slow cluster, it slows down and joins the cluster with probability $1 - W$. With probability W, it maintains its speed and passes the cluster. As we explained above, the probability for a car to undertake a passing maneuver depends on the velocity difference of the fast(overtaking) and slow(blocking) clusters. In the multi–agent terminology it means that an agent is more likely to join a cluster with velocity close to its own. To incorporate this assumption into the model we adopt the following simple functional form for the passing probability.

$$W(v - v') = \Theta(v - v' - V_R), \tag{1}$$

where $\Theta(x)$ is the step function, and V_R is a parameter of a model that determines the balance between an agent's desire for safety (not passing and joining the platoon) and minimizing its travel time (passing and keeping its inherent velocity). In the no passing limit $V_R \to \infty$ fast cars would cluster behind slower ones, which will bring the system to a congested phase at sufficiently high densities. In the opposite limit $V_R \to 0$ passing occurs very frequently as each car tends to optimize its own performance by maintaining its inherent velocity. In this paper we assume that V_R is the same for all agents, so the only source of the heterogeneity (disorder) is the initial velocity distribution $P_0(v)$.

Let $P_m(v,t)$ be the time–dependent density of clusters of size m and moving with velocity v, so that $mP_m(v)$ gives the probability that a car picked at random moves with velocity v in a cluster of size m. Also, let $U(v,v')$ be the rate at which cars (clusters) with velocity v are joining the cars (clusters) with velocity v'. For the Boltzman model from the kinetic theory of gases one has

$$U(v,v') = |v - v'|(1 - W(|v - v'|)) \tag{2}$$

The term $|v - v'|$ is simply the rate at which clusters with velocity v and v' approach each other, and $(1 - W(|v - v'|))$ is the probability that they will merge, as explained above.

The Master equation governing the time evolution of the joint size-velocity distribution reads:

$$\begin{aligned}\frac{\partial P_m(v)}{\partial t} &= -P_m(v)\sum_k \int_0^\infty dv' P_k(v')U(v,v') + \sum_{k+j=m}\int_v^\infty dv' P_k(v')P_j(v)U(v,v') \\ &+ \gamma[(m+1)P_{m+1}(v) - mP_m(v)] + \gamma P_0(v)\delta_{m,1},\ m = 1,2,3...\end{aligned} \tag{3}$$

The first term in Eq(3) describes the loss in the cluster density due to collision with other clusters, while the second term describes the gain in the cluster density due to the "merging" of two smaller clusters. Also, the terms proportional to γ describe the change in cluster density due to the inflow and outflow of vehicles on the freeway. One can check that the Eq.(3) conserves the car density, $\sum_k k \int_0^\infty dv P_k(v,t) = 1$.

The Master equation (3) is a complicated system of coupled integro–differential equations. In Ref.[3] authors studied the system(3) with velocity–independent collision rates (Maxwell model), $U(v,v') = const$, which they showed to be analytically tractable. With a velocity–dependent kernel (2), however, Eqs. (3) can not be solved analytically. Below we present some preliminary results obtained by numerical methods.

In Fig. 1 we plot the steady state cluster–size distribution $P_m = \int_0^\infty dv P_m(v, t \to \infty)$ for an initial uniform velocity distribution and with $\gamma = 0.001$, $V_R = 0.3$. One can see that the initial δ–like distribution has spread out. Note also, that P_m is not normalized since the total cluster density is not constant, in contrast to the car density. In fact, the cluster density is the inverse of the average cluster size: $< m >= \sum_k kP_k(t)/\sum_k P_k(t) = 1/\sum_k P_k(t)$. The time dependence of the average cluster size is illustrated in Fig. 2.

Finally, Fig.3 illustrates the steady state car velocity distribution. The initial homogenous distribution (dashed line) has evolved into multi–modal distribution, indi-

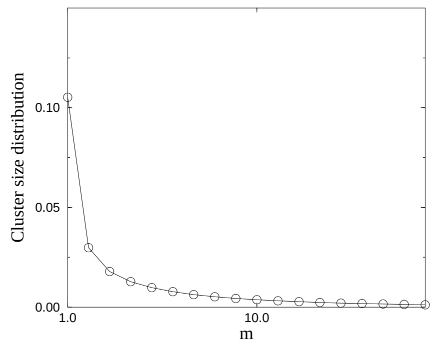

Figure 1: Steady state cluster–size distribution.

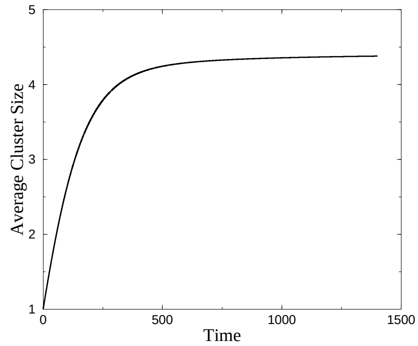

Figure 2: Average cluster size vs time.

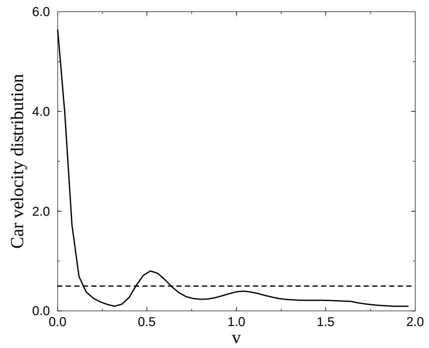

Figure 3: Car velocity distribution at steady state

cating that along with clusters formed behind the slowest cars, there are also faster clusters. This is due to the ”filtering” effect of the velocity–dependent passing probabilities. Note also, that the spread of the car velocities inside a cluster can be considerably greater than V_R. This is because a fast car might slow down several times due to further coalescence of clusters.

3 Discussion and Future Work

Clearly, the mechanism for platoon formation examined here is a very simple one. Since the model considered here is effectively a one–lane model, it does not take into account the restriction on passing caused by the cars in the other lane. In a more realistic model of multi–lane traffic, one should introduce cluster densities for each lane, $P_m^i, i = 1, 2 \ldots$. The appropriate Master equation will allow for coupling between different lanes. In particular, the probability for a cluster to pass a slower one will now strongly depend on its size. This will make the dynamics of the system more complex and interesting.

Another problem not addressed in this paper is the dependence of the total flow on the parameter V_R. It is clear intuitively that there is a V_R that optimizes the global performance of the system (i.e., the total flow, average travel time). Obviously, the optimal V_R will depend on the density of cars on the freeway. Note also, that one can introduce additional disorder by allowing agents to have different V_R.

Complementary to our phenomenological analysis of platoon formation, we intend to carry out microscopic simulation in the framework of a stochastic, particle–hoping model for two–lane freeway traffic. The freeway in this microscopic model is represented by a $2 \times L$ lattice where each site can be either empty or occupied by a single car (agent) and where cars move from left to right according to their (inherent) hopping probabilities drawn from a given distribution $P_0(v)$. This type of model is equivalent to a stochastic cellular automaton with random breaking probabilities and $v_{max} = 1$. Similar model has been studied by Nagatani[4].

References

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