

# A Geometric Method for Center of Mass Estimation in Rough Planar Terrains

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**Abstract**—Center of Mass(CoM) estimation in rough terrains is hampered by complicated body dynamics yet remains critically important in the study of human and robot motion planning. Current techniques for CoM estimation are encumbered by lengthy calibration periods requiring the use of specialized tools(force plates, motion capture, etc). This paper presents a novel and straightforward geometric method for CoM estimation over rough planar terrains that relies solely on geometry information of the environment and essential knowledge of the kinematic body. The CoM is approximated using a simplified model of the contact foot locations and an Optimized Geometric Hermite(OGH) curve with minimum curvature and strain energy(SE). To evaluate the accuracy of the method, cross validation with human subjects was performed. The results demonstrate that the geometric method delivers an accurate approximation of the CoM path for natural walking over rough planar terrains and offers a reliable alternative for CoM estimation.

## I. INTRODUCTION

The critical element in robot and human motion planning revolves around the ability to accurately measure and describe the CoM. Accurate CoM measurements are essential in understanding the behavior of a system, for example, in gait selection or in extreme locomotion maneuvers [1] [2]. In robotic systems control of the CoM plays a vital role in planning foot placements and maintaining static balance [3]. Due to the desire for humanoid robots to possess the high agility of their human counterparts, CoM estimation has become crucial in robot motion planning. Thus several techniques for CoM estimation in humans have been developed to shed light into its behavior.

The most direct method for CoM estimation is the body segmental approach. Although cumbersome to perform, the method has become the standard over the years due to its precision. The method relies on the use of anthropomorphic tables [4] [5] [6] to approximate the mass, position and orientation of the body segments for a subject. From these, an estimate of the whole-body CoM location can be produced. Although effective, the segmental method has several disadvantages. Specifically, the heavy reliance on continuous body tracking for prolonged measurement gathering renders the method impractical. Additionally, the method requires the use of motion capture devices which makes it infeasible in uncontrolled environments [7].

To relieve the dependency on motion capture data, other less obstructive data gathering methods have been introduced. In [8] a genetic sum of sines model for CoM trajectory

estimation was developed wherein the video capture device was replaced with two accelerometers. Force platforms for measuring ground reaction forces were used with Newton's second law to estimate the CoM in [9] [10] with treatment of the ambulatory case presented in [11]. An oscillating model and force plate recordings were also used to explore the relationship between the CoM and center of pressure(CoP) in [12] [13]. In general however, the constant reliance on CoP data provided by force platforms, or in the case of [11] awkward instrumented shoes, continues to make these methods unsuitable for non-laboratory settings.

Recent work has attempted to scupper the usage of extraneous measurement tools, either by increasing their portability or through the inclusion of an initial calibration process. The statically equivalent serial chain(SESC) method developed by Cotton et al. [14] [15] and first proposed by Espiau and Boulic [16] has proven a reliable method. The SESC method represents the CoM of an articulated body through a multi-link branched chain whose end-effector specifies the CoM location. The process involves a lengthy initial calibration period during which motion capture devices, force platform data and neutrally stable postures are used to reconstruct the subject's SESC. Once complete, only anatomical joint measurements are needed for CoM estimation. Following this work, the authors of [17]–[19] address the issue of tool cost and portability by resorting to a Kinect and Wii balance board to obtain visual and force data. Yet despite the incremental improvements, the methods discussed above all suffer from the same drawbacks, namely: the use of external instruments that are cost prohibitive and non-readily available, and data collection periods that are unworkable in non-laboratory settings.

The work presented in this paper offers an unobtrusive and attractive alternative for CoM estimation that is devoid of force or video sensors. The research is inspired by the idea that a body moving in an environment is channeled and guided by the restrictions placed on it by the terrain and the limitations inherent in the body's kinematic configuration. Using information of the contact foot locations, the terrain geometry and the kinematic constraints of the body, a simplified model is derived from which accurate CoM estimates are produced. The paper is outlined as follows: Section II provides a brief overview of the method followed by detailed explanations of each feature. Section III describes construction of the final CoM path over the terrain. Lastly, Section IV provides cross-validation with body segmental data for human subjects and comparisons and conclusions are drawn regarding the accurateness of the CoM estimate.

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## II. OVERVIEW OF METHOD

The method for producing CoM estimates for natural walking over rough planar terrains (Fig. 2) is as follows: (1) knowledge of the terrain, desired contact foot locations and step durations are used to create *virtual steps*. Virtual steps are nonphysical steps in the terrain that serve as intermediate points during double support phases. (2) Virtual steps, desired contact foot locations, and the geometry of the terrain are used to define forward progress angles. Forward progress angles are simply the direction of motion of the CoM at the apex of each step (virtual and real). (3) An OGH curve with minimum curvature and strain energy is used in conjunction with basic body kinematic parameters to construct the CoM path. This is accomplished by incrementally building the CoM path between steps using the forward progress angles. The composition of each piecewise OGH segment between steps results in the complete CoM path through the terrain.

### A. Virtual Steps

Virtual steps are immaterial reference points located in the terrain that describe CoM behavior between foot locations. Specifically, virtual steps represent the location and amount of *dip* that occurs when the ensuing foot comes into contact with the terrain. The idea from comes the observation that as a subject performs the next step, a negative vertical displacement occurs in the CoM location relative to its stationary upright position. This displacement  $\Delta_{com_z}$  can be used to describe the behavior of the CoM between CoPs. Thus, a virtual step is defined as a point in space  $(x_{vs}, z_{vs})$  lying in between sequential contact foot locations whose value is derivable from adjacent CoPs. This paper employs a geometric method and treats the three points, hip joint center and successive CoPs, as forming a moving and varying isosceles triangle. The height  $h$  of the triangle is then used to find the displacement  $\Delta_{com_z}$ . This geometric representation is shown in Fig. 1 where  $b$  is the leg length,  $d$  is the distance from the hip joint center to the subject CoM in the upright posture as given in [4] [6], and  $a$  is the horizontal distance between CoPs. Thus, given two CoPs  $(x_{cop_i}, z_{cop_i})$  and  $(x_{cop_j}, z_{cop_j})$  in the terrain, the CoM vertical displacement  $\Delta_{com_z}^{ij}$  is given by

$$\Delta_{com_z}^{ij} = (b + d) - (h_{ij} + d) \quad (1)$$

which expanding terms gives

$$\Delta_{com_z}^{ij} = b - \sqrt{b^2 - \frac{(x_{cop_j} - x_{cop_i})^2}{4}} \quad (2)$$

The vertical component of the virtual step  $z_{vs}^i$  is then selected to be

$$z_{vs}^i = \min(z_{cop_i}, z_{cop_j}) - \Delta_{com_z}^{ij} \quad (3)$$

The horizontal value  $x_{vs}^i$  of a virtual step is calculated by considering both the height difference between adjacent CoPs and the duration of each foot contact. Given desired CoP durations  $\Delta t_i$  and  $\Delta t_j$  for two sequential steps  $(x_{cop_i}, z_{cop_i})$  and  $(x_{cop_j}, z_{cop_j})$ , the time and height

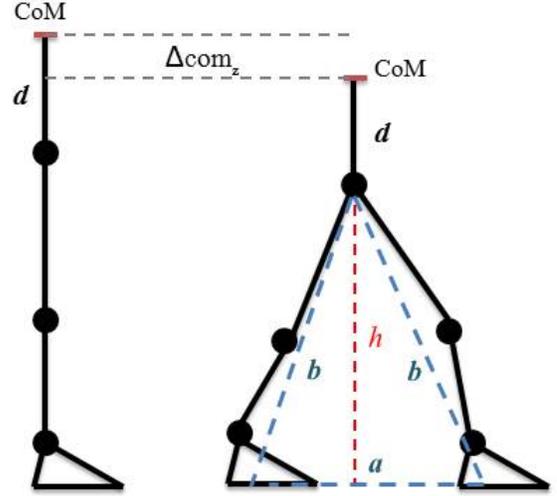


Fig. 1. Geometric representation of step motion on sagittal plane.

proportions of  $x_{vs}^i$  are defined as

$$T = \frac{\max(\Delta t_i, \Delta t_j)}{\Delta t_i + \Delta t_j} \quad (4)$$

$$H = \begin{cases} \frac{\max(z_{cop_i}, z_{cop_j})}{z_{cop_i} + z_{cop_j}} & \text{if } z_{cop_i} \neq z_{cop_j} \\ 0.5 & \text{if } z_{cop_i} = z_{cop_j} \end{cases} \quad (5)$$

The horizontal displacement for  $x_{vs}^i$  is formed as follows

$$\Delta_x^{ij} = |x_{cop_i} - x_{cop_j}| \left( \frac{T + H}{2} \right) \quad (6)$$

and  $x_{vs}^i$  is chosen as

$$x_{vs}^i = \begin{cases} x_{cop_i} + \Delta_x^{ij} & \text{if } \Delta t_i \geq \Delta t_j \\ x_{cop_j} - \Delta_x^{ij} & \text{if } \Delta t_i < \Delta t_j \end{cases} \quad (7)$$

Thus, virtual step locations are mediatory points for double support phases and are guaranteed to lie between consecutive CoPs and have vertical values  $z_{vs}^i$  below  $\min(z_{cop_i}, z_{cop_j})$ .

### B. Forward Progress Angles

As previously mentioned, forward progress angles specify the direction of motion of the CoM at the apex of each step, real and virtual. The angles are found through the observation that CoM behavior at the current step is closely related to the location and distance of the current step to both the preceding and ensuing foot steps. Because the steps alternate between real and virtual, knowing desired CoP and virtual step locations provides an angular description of CoM behavior through the terrain. This is accomplished through an arctan angle calculation between steps. In general, virtual step angles are defined with respect to the previous real step CoP, while real step angles are defined based on the proximity and height location of the ensuing real CoP. The complete algorithm is presented in Algorithm 1 while an example terrain with virtual steps, foot contact CoPs and forward angles is demonstrated in Fig. 2.

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**Algorithm 1:** Forward Progress Angles

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**Input:** Let  $S=\{1..n\}$  be the steps with  $S$  odd being real steps and  $S$  even the virtual steps.  
 $\psi_{ij} \leftarrow \text{atan2d}$  angle from step  $(x_i, z_i)$  to  $(x_j, z_j)$ .  
 $\theta_i \leftarrow$  forward angle of each step.  
Initial angle  $\theta_1$  for  $S=\{1\}$  is defined as

$$\theta_1 = \begin{cases} \max(|\psi_{12}|, \psi_{13}) & \text{if } z_3 \geq z_1 \\ \min(|\psi_{12}|, |\psi_{13}|) & \text{if } z_3 < z_1 \end{cases}$$

**for**  $i = 2$  to  $n$  **do**

**if**  $S=\{i\}$  is a virtual step **then**

$$\theta_i = \psi_{i-1,i}$$

**else** {real step}

$$\theta_i = \begin{cases} \max(\psi_{i-1,i}, \psi_{i,i+2}) & \text{if } z_{i+2} \geq z_i \\ \psi_{i-1,i} & \text{if } z_{i+2} < z_i \end{cases}$$

**end if**

**end for**

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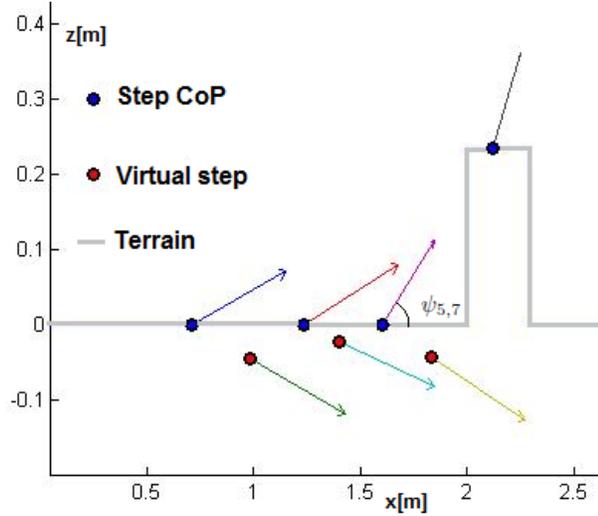


Fig. 2. Four step walking sequence through terrain. The virtual steps and forward angles(vectors) are also shown.

### C. Optimized geometric Hermite curve

An optimized geometric Hermite curve is a piecewise spline specified in Hermite form and optimized for minimum length, strain energy or curvature [20]–[22]. This paper uses OGH curves with minimum curvature first defined in [22] as the basis for CoM estimation since they deliver geometrically smooth curves free of extreme irregularities. Additionally, the ability to specify endpoint tangent vectors allows for directional control giving greater command of curve creation.

**Definition 1.** Given two endpoints  $P_0$  and  $P_1$ , and two endpoint tangent vectors  $V_0$  and  $V_1$ , a cubic polynomial curve  $P(t)$ ,  $t \in [t_0, t_1]$ , is called an optimized geometric Hermite(OGH) curve with respect to the endpoint conditions  $P_0, P_1, V_0, V_1$  if it has the smallest curvature variation among all cubic Hermite curves  $\bar{P}(t)$ ,  $t \in [t_0, t_1]$  satisfying the following condition:

$$\bar{P}(t_0) = P_0, \bar{P}(t_1) = P_1, \bar{P}'(t_0) = \alpha_0 V_0, \bar{P}'(t_1) = \alpha_1 V_1 \quad (8)$$

where  $\alpha_0$  and  $\alpha_1$  are arbitrary real numbers, and the cubic Hermite curve  $\bar{P}(t)$ ,  $t \in [t_0, t_1]$  satisfying the constraints in (8) can be expressed as

$$\bar{P}(t) = (2s+1)(s-1)^2 P_0 + (-2s+3)s^2 P_1 + (1-s)^2 s(t_1-t_0)\alpha_0 V_0 + (s-1)s^2(t_1-t_0)\alpha_1 V_1 \quad (9)$$

where  $s = (t-t_0)/(t_1-t_0)$ . The objective function to optimize is defined as

$$E = \int_{t_0}^{t_1} [\bar{P}'''(t)]^2 dt \quad (10)$$

and is the approximate curvature variation of the curve  $\bar{P}(t)$ .

Tangent angles  $\theta$  and  $\phi$  are used to define tangent vectors  $V_0$  and  $V_1$  respectively, with  $\theta$  the counterclockwise angle from the vector  $\bar{P}_0\bar{P}_1$  to  $V_0$ , and  $\phi$  the counterclockwise angle from the vector  $\bar{P}_0\bar{P}_1$  to  $V_1$ . This work uses the forward progress angles as the tangent angle values. Using forward progress angles produces OGH curves that heed the direction of motion of the CoM and yields behavior that adheres to step locations and the natural motion over the terrain. For details regarding OGH curves and tangent angles, see [22].

### III. CONSTRUCTION OF CoM PATH

The process for producing CoM estimates is as follows: 1) two step locations and forward progress angles are used to generate OGH curves with minimum curvature. The OGH curves must satisfy the body's kinematic constraints. 2) The OGH curve having minimum (SE) is selected as the CoM path. The endpoint of the path defines the start point for the next iteration. 3) The next step and forward angle is taken, and the process continues until the final step is included.

This paper abstracts body kinematic constraints into an interval range  $\{(z_{min}+k, z_{max}+k) | (z_{min}+k) \leq p_{com_z} \leq (z_{max}+k)\}$ , which represents the space of realizable CoM positions for a specific body at height  $k$  of the terrain. This allows us to characterize the kinematic variation present in robots and humans and generate paths that respect the limits of the kinematic architecture. Note that for efficiency the range is discretized into intervals of 0.01m. Furthermore, only planar environments are treated leading  $(z_{min}+k, z_{max}+k)$  to depend on the terrain such that  $\forall p_{ter_z} \in \mathbb{R} : |(z_{min}+k) - p_{ter_z}| = c$ , where  $p_{ter_z}$  is the vertical position of the terrain and  $c$  is a constant. Thus, the body's kinematic constraints sets a lower bound on viable CoM positions.

For human subjects,  $z_{min}$  and  $z_{max}$  are approximated directly from body segmental data. Using the subject's height,  $z_{max}$  is estimated as the upright standing posture CoM location plus the foot heel to ball distance, while  $z_{min}$  is set to the vertical CoM position during performance of a lunge. Note that only height information and mean segment lengths [4] [23] are required greatly facilitating the computation of subject specific kinematic constraints. Furthermore, because the CoM paths are generated step-to-step, the vertical location of each step  $(x_i, z_i)$  identifies the value for  $k$ . Perforce this allows for production of CoM paths that follow the locomotion of a subject through the terrain. The algorithm for generating CoM paths is given in Algorithm 2 with an example presented in Fig. 3.

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**Algorithm 2: CoM Path Construction**

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**Input:**Steps  $S = \{(x_1, z_1) \dots (x_n, z_n)\}$ Forward angles  $\{\theta_1 \dots \theta_n\}$ Constraints  $z_{min}$  and  $z_{max}$ Initial CoM position  $(x_1, p_{com_z, init})$ **for**  $i = 2$  to  $n$  **do****for**  $p_{com_z} = (z_{min} + z_i)$  to  $(z_{max} + z_i)$  **do**

1. Generate OGH curve with minimum curvature

using  $(x_1, p_{com_z, init})$ ,  $\theta_1$  to  $(x_i, p_{com_z})$ ,  $\theta_i$ 

2. If OGH curve violates CoM lower bound threshold, reject curve

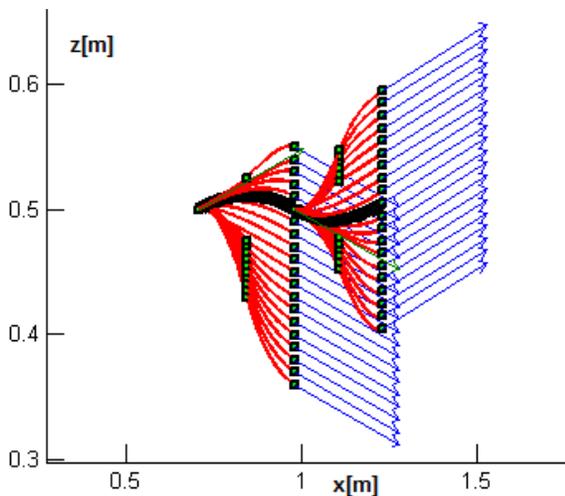
3.  $p_{com_z} = p_{com_z} + 0.01m$ **end for**a. Select endpoint  $(x_i, p_{com_z})$  corresponding to curve  $G_\ell$  with minimum (SE) among all curves generated.b. Set CoM path to  $G_\ell$ c.  $x_1 = x_i$ ,  $p_{com_z, init} = p_{com_z}$ ,  $\theta_1 = \theta_i$ **end for****return** Complete CoM path, the piecewise composition of each curve  $G_\ell$ .

Fig. 3. Example OGH curves (red) for first three steps and forward angles of Fig. 2. At each step, the curve with minimum (SE) is selected as the CoM path (black).

## IV. EXPERIMENTAL RESULTS

### A. Experimental Setup and Data Collected

Six subjects were asked to traverse two different terrains, each consisting of variable height obstacles. The human traversal data was collected using Matlab and the motion capture method with the video camera placed sufficiently far from the subjects so as to render the recordings of pure planar motion with negligible lens distortion. The motion capture video was taken at 30 frames per second, however, to reduce data processing, only every 3<sup>rd</sup> frame was used. The locations of the CoMs of 14 individual body segments [4] were used with markers placed: 1 on each foot, calf, thigh, upper arm, forearm and hand, plus one for the torso (pelvis,

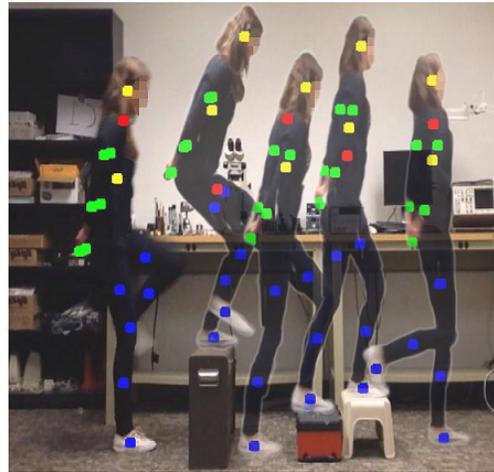


Fig. 4. Sagittal motion capture of human traversal over terrain.

TABLE I:  
Subject Characteristics Data.

Subject	Gender	Height	Weight
1	M	1.75m	68kg
2	M	1.75m	84kg
3	F	1.73m	56kg
4	M	1.72m	75kg
5	M	1.78m	82kg
6	F	1.55m	41kg

abdomen and chest) and 1 for the head (head and neck). The subject's CoM location for each run was calculated using a vectorial weighted sum of body segment CoM locations and their relative masses with Matlab's `cftool` function used to generate a best fit curve for the data. The supporting point of contact for each step  $p_{cop_k}$  was obtained directly from Matlab's `ginput` data and initially estimated as the midpoint of the surface foot contact location, while step durations were obtained from the video data. Fig. 4 shows a sample traversal while subject characteristics are reported in Table I.

### B. Cross-validation

CoM estimates were automatically generated for each subject using the geometric method and comparisons made with the motion capture CoM data. The automatic curves were generated with a  $\pm 5cm$  x-direction adjustment to  $p_{cop_k}$ . This was observed to produce better correlations with the measured data and is a valid adjustment in light of the impossibility of humans to walk with true point contact behavior due to ankle use, foot structure etc. Thus, the CoP of the system will change constantly and cannot be measured using our motion capture technique. Fig. 5 and 6 show the measured (motion capture) and geometrically estimated CoM paths for two subjects over both terrains and demonstrates the similitude between the measured and estimated paths. The root mean squared error (rmse) between the vertical measured and estimated CoM position for all 12 runs was also calculated and is given in Table II.

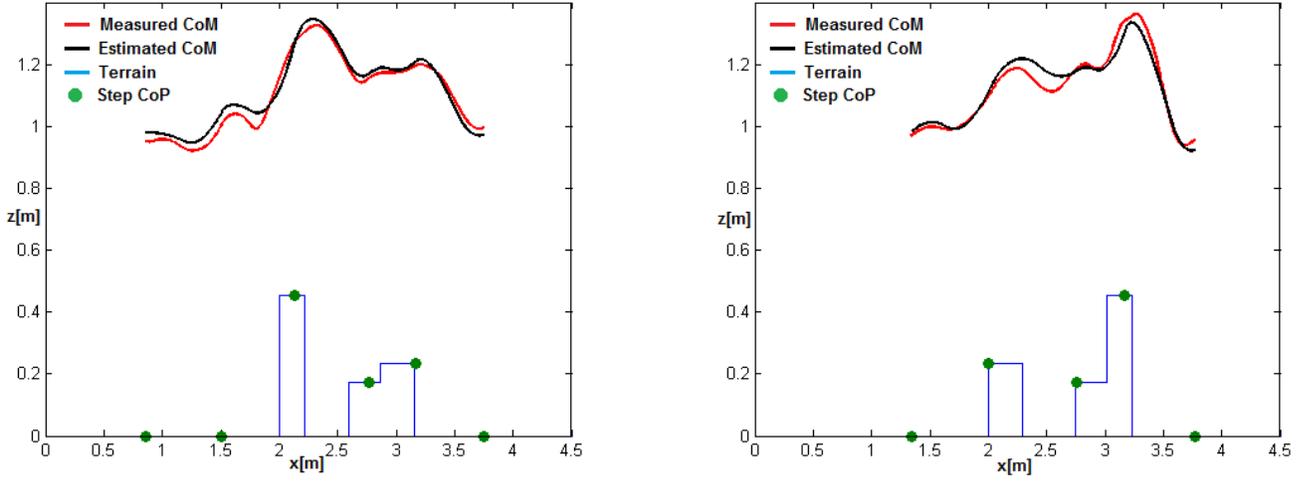


Fig. 5: Subject 4 performing a 6 step walking sequence over terrain 1(left) and a 5 step walking sequence over terrain 2(right). The rmse of estimated versus measured CoM for each was 0.0242m and 0.0285m respectively.

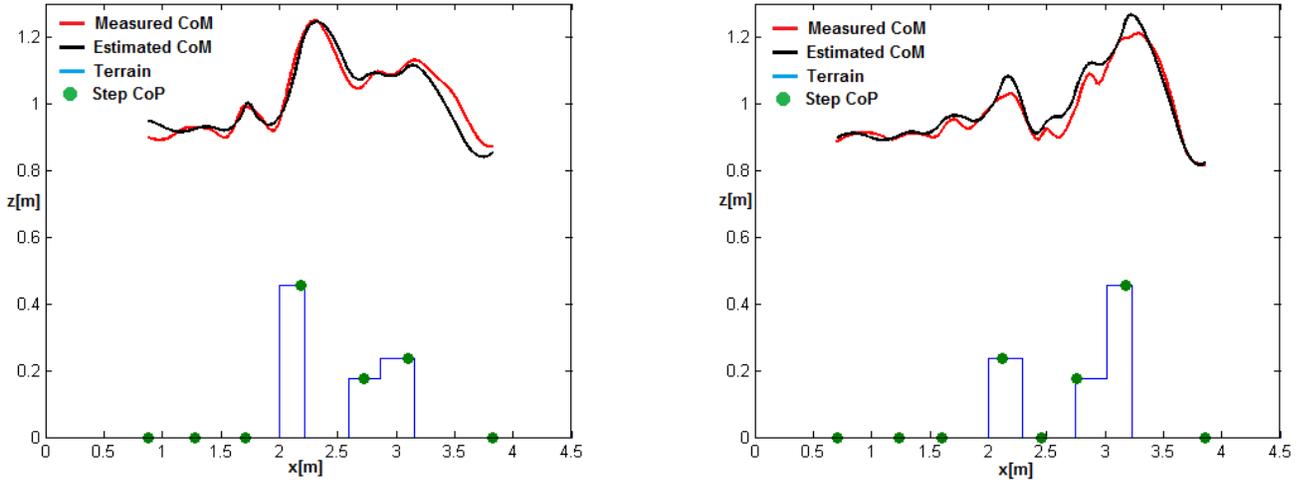


Fig. 6: Subject 6 performing a 7 step walking sequence over terrain 1(left) and an 8 step walking sequence over terrain 2(right). The rmse of estimated versus measured CoM for each was 0.0319m and 0.0272m respectively.

TABLE II:  
RMSE Est. vs Measured CoM Vertical Position

Subject	Terrain 1	Terrain 2
1	0.0229m	0.0475m
2	0.0273m	0.0305m
3	0.0364m	0.0273m
4	0.0242m	0.0285m
5	0.0579m	0.0444m
6	0.0319m	0.0272m
avg rmse	0.0334m	0.0342m

The average rmse for all subjects over both terrains was 0.0334m and 0.0342m, demonstrating a striking similarity in the results across different terrains. Furthermore, it is

encouraging to note that the accuracy of the results obtained were for subjects possessing varying heights and weights, utilizing different step amounts(e.g. the 5 step sequence of Fig. 5(right) and the 8 step sequence of Fig. 6(right)) and traveling at different speeds. This is an important observation to emphasize since the validity of the geometric method for natural walking over rough planar terrains must endure irrespective of the body kinematic constraints, foot contact locations, or the terrain traversed. Even in cases displaying higher rmse, such as subject 5 and terrain 1, the results indicated a strong correlation in the CoM estimate produced by the geometric method and the motion capture data. In general, results between estimation and measurements from motion capture data were very close and displayed consistent accuracy among all test subjects. The propitious

results gathered using a variety of terrains, subjects, and walking sequences asserts that despite the simplicity of the approach, the geometric method was capable of generating CoM estimates that closely reproduced the CoM pattern of behavior performed by the human subjects.

## V. DISCUSSION AND FUTURE WORK

This paper presented a simple and general technique for CoM estimation over rough planar terrains bereft of external instrumentation. The geometric method allows for CoM estimation of natural walking locomotion from essential kinematic parameters and the geometry of the terrain without resorting to motion capture or force platform data.

The novel approach proposed here offers several advantages. Firstly, the non-reliance on motion capture or force measurement hardware renders the method immensely practical and portable. Such an advantage obviates the need for costly and cumbersome devices requiring time consuming and uncouth calibration periods. This greatly facilitates the estimation of subject specific CoM paths. Secondly, the successful application of this method to rough planar terrains makes the geometric approach suitable for a wide range of test environments, increasing its scope and versatility. Lastly, the method requires minimal input information regarding the subject and the terrain, greatly reducing the complexity of its usage.

The CoM estimate obtained compared with the motion capture data produced small rmse, nonetheless, further work is necessary to improve its accuracy. Specifically, investigation of more disparate subjects should be explored to determine how they affect virtual step locations and forward progress angle calculations. This could lead to finer optimizations that address and account for the heterogeneity of large populations. Expansion to more complex terrains could also be examined, perhaps leading to further refinements in the method. Overall, the geometric approach produced accurate CoM estimates of human locomotion over rough planar terrains and further improvements could increase the functional scope of the method and build upon the work undertaken thus far.

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