Toward an Ontology of Time for the Semantic Web

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Abstract

In connection with the DAML project for bringing about the Semantic Web, an ontology of time is being developed for describing the temporal content of Web pages and the temporal properties of Web services. The bulk of information on the Web is in natural language, and this information will be easier to encode for the Semantic Web insofar as community-wide annotation and automatic tagging schemes and the DAML time ontology are compatible with each other.

1. Introduction

The DARPA Agent Markup Language (DAML) project is an effort aimed at bringing into reality the Semantic Web, in which Web users and automatic agents will be able to access information on the Web via descriptions of the content and capabilities of Web resources rather than via key words. An important part of this effort is the development of representative ontologies of the most commonly used domains. We are beginning to develop such an ontology of temporal concepts, for describing the temporal content of Web pages and the temporal properties of Web services. This effort is being informed by temporal ontologies developed at a number of sites and is intended to capture the essential features of all of them and make them easily available to a large group of Web developers and users.

The bulk of information on the Web is in natural language, and this information will be easier to encode for the Semantic Web insofar as community-wide annotation and automatic tagging schemes and the DAML time ontology are compatible with each other.

In this paper I outline the temporal ontology as it has been developed so far, in order to initiate a dialog between the two communities. Five categories of temporal concepts are considered, and for each the principal predicates and their associated properties are described.

A note on notation: Conjunction ($\wedge$) takes precedence over implication($\supset$) and equivalence ($\equiv$). Formulas are assumed to be universally quantified on the variables appearing in the antecedent of the highest-level implication. Thus,

$$p_1(x) \wedge p_2(y) \supset q_1(x,y) \wedge q_2(y)$$

is to be interpreted as

$$(\forall\, x,y)[[p_1(x) \wedge p_2(y)] \supset [q_1(x,y) \wedge q_2(y)]]$$

2. Topological Temporal Relations

2.1. Instants and Intervals

There are two subclasses of temporal-entity: $instant$ and $interval$.

$$instant(t) \supset temporal\text{-}entity(t)$$
$$interval(T) \supset temporal\text{-}entity(T)$$

(In what follows, lower case t is used for instants, upper case T for intervals and for temporal-entities unspecified as to subtype. This is strictly for the reader's convenience, and has no formal significance.)

$start\text{-}of$ and $end\text{-}of$ are functions from temporal entities to instants.

$$temporal\text{-}entity(T) \supset instant(start\text{-}of(T))$$
$$temporal\text{-}entity(T) \supset instant(end\text{-}of(T))$$

For convenience, we can say that the start and end of an instant is itself.

$$instant(t) \supset start\text{-}of(t) = t$$
$$instant(t) \supset end\text{-}of(t) = t$$

$inside$ is a relation between an instant and an interval.

$$inside(t,T) \supset instant(t) \wedge interval(T)$$

This concept of $inside$ is not intended to include starts and ends of intervals, as will be seen below.

Infinite and half-infinite intervals can be handled by positing time instants at positive and negative infinity, and using them as start and end points.

It will be useful in characterizing clock and calendar terms to have a relation between instants and intervals that says that the instant is inside or the start of the interval.

$$in\text{-}interval(t,T) \equiv [start\text{-}of(T) = t \vee inside(t,T)]$$

$interval\text{-}between$ is a relation among a temporal entity and two instants.

$$interval\text{-}between(T,t_1,t_2) \supset temporal\text{-}entity(T) \wedge instant(t_1) \wedge instant(t_2)$$

The two instants are the start and end points of the temporal entity.

$$interval\text{-}between(T,t_1,t_2) \equiv start\text{-}of(T) = t_1 \wedge end\text{-}of(T) = t_2$$

The ontology is silent about whether the interval from t to t, if it exists, is identical to the instant t.

The ontology is silent about whether intervals *consist of* instants.

The ontology is silent about whether intervals are uniquely determined by their starts and ends.

We can define a proper interval as one whose start and end are not identical.

$$proper\text{-}interval(t) \equiv \\ interval(t) \wedge start\text{-}of(t) \neq end\text{-}of(t)$$

The ontology is silent about whether there are any intervals that are not proper-intervals.

2.2. Before

There is a $before$ relation on temporal entities, which gives directionality to time. If temporal entity T_1 is before temporal entity T_2, then the end of T_1 is before the start of T_2. Thus, before can be considered to be basic to instants and derived for intervals.

$$before(T_1, T_2) \\ \equiv before(end\text{-}of(T_1), start\text{-}of(T_2))$$

The end of an interval is not before the start of the interval.

$$interval(T) \\ \supset \neg before(end\text{-}of(T), start\text{-}of(T))$$

The start of a proper interval is before the end of the interval.

$$proper\text{-}interval(T) \\ \supset before(start\text{-}of(T), end\text{-}of(T))$$

If one instant is before another, there is an interval between them.

$$instant(t_1) \wedge instant(t_2) \wedge before(t_1, t_2) \\ \supset (\exists T) interval\text{-}between(T, t_1, t_2)$$

The ontology is silent about whether there is an interval from t to t.

If an instant is inside a proper interval, then the start of the interval is before the instant, which is before the end of the interval. The converse is true as well.

$$instant(t) \wedge proper\text{-}interval(T) \\ \supset [inside(t, T) \\ \equiv before(start\text{-}of(T), t) \\ \wedge before(t, end\text{-}of(T))]$$

Intervals are contiguous with respect to the $before$ relation, in that an instant between two other instants inside an interval is inside the interval.

$$before(t_1, t_2) \wedge before(t_2, t_3) \\ \wedge inside(t_1, T) \wedge inside(t_3, T) \\ \supset inside(t_2, T)$$

The $before$ relation is anti-symmetric and transitive.

$$before(T_1, T_2) \supset \neg before(T_2, T_1)$$

$$before(T_1, T_2) \wedge before(T_2, T_3) \\ \supset before(T_1, T_3)$$

The relation $after$ is defined in terms of $before$.

$$after(T_1, T_2) \equiv before(T_2, T_1)$$

The ontology is silent about whether time is linearly ordered.

2.3. Interval Relations

The relations between intervals defined in Allen's temporal interval calculus (Allen and Kautz, 1985) can be defined in a straightforward fashion in terms of $before$ and identity on the start and end points.

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}equals(T_1, T_2) \\ \equiv start\text{-}of(T_1) = start\text{-}of(T_2) \\ \wedge end\text{-}of(T_1) = end\text{-}of(T_2)]$$

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}before(T_1, T_2) \equiv before(T_1, T_2)$$

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}after(T_1, T_2) \equiv after(T_1, T_2)$$

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}meets(T_1, T_2) \\ \equiv end\text{-}of(T_1) = start\text{-}of(T_2)$$

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}met\text{-}by(T_1, T_2) \\ \equiv int\text{-}meets(T_2, T_1)]$$

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}overlaps(T_1, T_2) \\ \equiv before(start\text{-}of(T_1), start\text{-}of(T_2)) \\ \wedge before(start\text{-}of(T_2), end\text{-}of(T_1)) \\ \wedge before(end\text{-}of(T_1), end\text{-}of(T_2))]$$

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}overlapped\text{-}by(T_1, T_2) \\ \equiv int\text{-}overlaps(T_2, T_1)]$$

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}starts(T_1, T_2) \\ \equiv start\text{-}of(T_1) = start\text{-}of(T_2) \\ \wedge before(end\text{-}of(T_1), end\text{-}of(T_2)]$$

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}started\text{-}by(T_1, T_2) \\ \equiv int\text{-}starts(T_2, T_1)]$$

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}during(T_1, T_2) \\ \equiv (before(start\text{-}of(T_2), start\text{-}of(T_1)) \\ \wedge before(end\text{-}of(T_1), end\text{-}of(T_2))]$$

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}contains(T_1, T_2) \\ \equiv int\text{-}during(T_2, T_1)]$$

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}finishes(T_1, T_2) \\ \equiv before(start\text{-}of(T_2), start\text{-}of(T_1)) \\ \wedge end\text{-}of(T_1) = end\text{-}of(T_2)]$$

$$interval(T_1) \wedge interval(T_2) \\ \supset [int\text{-}finished\text{-}by(T_1, T_2) \\ \equiv int\text{-}finishes(T_2, T_1)]$$

In addition, it will be useful below to have a single predicate for "starts or is during". This is called $int\text{-}in$.

$$int\text{-}in(T_1, T_2) \\ \equiv [int\text{-}starts(T_1, T_2) \vee int\text{-}during(T_1, T_2)]$$

It will also be useful to have a single predicate for intervals intersecting in at most an instant.

$$int\text{-}disjoint(T_1,T_2)$$
$$\equiv [int\text{-}before(T_1,T_2) \vee int\text{-}after(T_1,T_2)$$
$$\vee\, int\text{-}meets(T_1,T_2)$$
$$\vee\, int\text{-}met\text{-}by(T_1,T_2)]$$

So far, the concepts and axioms in the ontology of time would be appropriate for scalar phenomena in general.

2.4. Linking Time and Events

The time ontology links to other things in the world through four predicates—$at\text{-}time$, $during$, $holds$, and $time\text{-}span\text{-}of$. We assume that another ontology provides for the description of events—either a general ontology of event structure abstractly conceived, or specific, domain-dependent ontologies for specific domains.

The term "eventuality" will be used to cover events, states, processes, propositions, states of affairs, and anything else that can be located with respect to time. The possible natures of eventualities would be spelled out in the event ontologies.

The predicate $at\text{-}time$ relates an eventuality to an instant, and is intended to say that the eventuality holds, obtains, or is taking place at that time.

$$at\text{-}time(e,t) \supset eventuality(e) \wedge instant(t)$$

The predicate $during$ relates an eventuality to an interval, and is intended to say that the eventuality holds, obtains, or is taking place during that interval.

$$during(e,T) \supset eventuality(e) \wedge interval(T)$$

If an eventuality obtains during an interval, it obtains at every instant inside the interval.

$$during(e,T) \wedge inside(t,T) \supset at\text{-}time(e,t)$$

Whether a particular process is viewed as instantaneous or as occuring over an interval is a granularity decision that may vary according to the context of use, and is assumed to be provided by the event ontology.

Often the eventualities in the event ontology are best thought of as propositions, and the relation between these and times is most naturally called $holds$. $holds$ can be defined in terms of $at\text{-}time$ and $during$:

$$holds(e,t) \wedge instant(t) \equiv at\text{-}time(e,t)$$
$$holds(e,T) \wedge interval(T) \equiv during(e,T)$$

The event ontology may provide other ways of linking events with times, for example, by including a time parameter in predications.

$$p(x,t)$$

This time ontology provides ways of reasoning about the t's; their use as arguments of predicates from another domain would be a feature of the ontology of the other domain.

The predicate $time\text{-}span\text{-}of$ relates eventualities to instants or intervals. For contiguous states and processes, it tells the entire instant or interval for which the state or process obtains or takes place.

$$time\text{-}span\text{-}of(T,e)$$
$$\supset temporal\text{-}entity(T) \wedge eventuality(e)$$
$$time\text{-}span\text{-}of(T,e) \wedge interval(T)$$
$$\supset during(e,T)$$
$$time\text{-}span\text{-}of(t,e) \wedge instant(t)$$
$$\supset at\text{-}time(e,t)$$

$$time\text{-}span\text{-}of(T,e) \wedge interval(T)$$
$$\wedge \neg inside(t_1,T) \wedge \neg start\text{-}of(t_1,T)$$
$$\wedge \neg end\text{-}of(t_1,T)$$
$$\supset \neg at\text{-}time(e,t_1)$$
$$time\text{-}span\text{-}of(t,e) \wedge instant(t) \wedge t_1 \neq t$$
$$\supset \neg at\text{-}time(e,t_1)$$

$time\text{-}span\text{-}of$ is a predicate rather than a function because until the time ontology is extended to aggregates of temporal entities, the function would not be defined for non-contiguous eventualities. Whether the eventuality obtains at the start and end points of its time span is a matter for the event ontology to specify. The silence here on this issue is the reason $time\text{-}span\text{-}of$ is not defined in terms of necessary and sufficient conditions.

The event ontology could extend temporal functions and predicates to apply to events in the obvious way, e.g.,

$$ev\text{-}start\text{-}of(e) = t$$
$$\equiv time\text{-}span\text{-}of(T,e) \wedge start\text{-}of(T) = t$$

This would not be part of the time ontology, but would be consistent with it.

Different communities have different ways of representing the times and durations of states and events (processes). In one approach, states and events can both have durations, and at least events can be instantaneous. In another approach, events can only be instantaneous and only states can have durations. In the latter approach, events that one might consider as having duration (e.g., heating water) are modeled as a state of the system that is initiated and terminated by instantaneous events. That is, there is the instantaneous event of the start of the heating at the start of an interval, that transitions the system into a state in which the water is heating. The state continues until another instantaneous event occurs—the stopping of the heating at the end of the interval. These two perspectives on events are straightforwardly interdefinable in terms of the ontology we have provided. This is a matter for the event ontology to specify. This time ontology is neutral with respect to the choice.

3. Measuring Durations

3.1. Temporal Units

This development assumes ordinary arithmetic is available.

There are at least two approaches that can be taken toward measuring intervals. The first is to consider units of time as functions from Intervals to Reals, e.g.,

$minutes$: Intervals $\rightarrow$ Reals
$minutes([5:14, 5:17]) = 3$

The other approach is to consider temporal units to constitute a set of entities—call it TemporalUnits—and have a single function $duration$ mapping Intervals × TemporalUnits into the Reals.

$$duration([5:14, 5:17], *Minute*) = 3$$

The two approaches are interdefinable:

$$\begin{array}{l} seconds(T) = duration(T, *Second*) \\ minutes(T) = duration(T, *Minute*) \\ hours(T) = duration(T, *Hour*) \\ days(T) = duration(T, *Day*) \\ weeks(T) = duration(T, *Week*) \\ months(T) = duration(T, *Month*) \\ years(T) = duration(T, *Year*) \end{array}$$

Ordinarily, the first is more convenient for stating specific facts about particular units. The second is more convenient for stating general facts about all units.

The aritmetic relations among the various units are as follows:

$$\begin{array}{l} seconds(T) = 60 * minutes(T) \\ minutes(T) = 60 * hours(T) \\ hours(T) = 24 * days(T) \\ days(T) = 7 * weeks(T) \\ months(T) = 12 * years(T) \end{array}$$

The relation between days and months (and, to a lesser extent, years) will be specified as part of the ontology of clock and calendar below. On their own, however, month and year are legitimate temporal units.

In this development durations are treated as functions on intervals and units, and not as first class entities on their own, as in some approaches. In the latter approach, durations are essentially equivalence classes of intervals of the same length, and the length of the duration is the length of the members of the class. The relation between an approach of this sort (indicated by prefix D-) and the one presented here is straightforward.

$$\begin{array}{l} (\forall\, T, u, n)[duration(T, u) = n \\ \quad \equiv\ (\exists\, d)[D\text{-}duration\text{-}of(T) = d \\ \qquad \wedge\, D\text{-}duration(d, u) = n]] \end{array}$$

At the present level of development of the temporal ontology, this extra layer of representation seems superfluous. It may be more compelling, however, when the ontology is extended to deal with the combined durations of noncontiguous aggregates of intervals.

3.2. $Hath$

The multiplicative relations above don't tell the whole story of the relations among temporal units. Temporal units are *composed of* smaller temporal units. The basic predicate used here for expressing the composition of larger intervals out of smaller clock and calendar intervals is $Hath$, from statements like "30 days hath September" and "60 minutes hath an hour." Its structure is

$$Hath(S, N, u, x)$$

meaning "A set S of N calendar intervals of type u hath the calendar interval x." That is, if $Hath(S, N, u, x)$ holds, then x is composed of the disjoint union of N intervals of type u; S is the set of those intervals. For example, if x is some month of September and S is the set of the successive days of that September, then $Hath(S, 30, *Day*, x)$ would be true.

The principal properties of $Hath$ are as follows:

The type constraints on its arguments: S is a set, N is an integer, u is a temporal unit, and x is an interval:

$$\begin{array}{l} Hath(S, N, u, x) \\ \quad \supset\ set(S)\ \wedge\ integer(N) \\ \qquad \wedge\, temporal\text{-}unit(u)\ \wedge\ interval(x) \end{array}$$

The elements of S are intervals of duration u:

$$\begin{array}{l} Hath(S, N, u, x) \\ \quad \supset\ (\forall\, y)[member(y, S) \\ \qquad \supset\ interval(y)\ \wedge\ duration(y, u) = 1] \end{array}$$

S has N elements:

$$Hath(S, N, u, x)\ \supset\ card(S) = N$$

The elements of S are disjoint:

$$\begin{array}{l} Hath(S, N, u, x) \\ \quad \supset\ (\forall\, y_1, y_2)[member(y_1, S) \\ \qquad\quad \wedge\, member(y_2, S)\ \wedge\ y_1 \neq y_2 \\ \qquad \supset\ int\text{-}disjoint(y_1, y_2)] \end{array}$$

There are elements in S that start and finish x:

$$\begin{array}{l} Hath(S, N, u, x) \\ \quad \supset\ (\exists\, y_1)[member(y_1, S) \\ \qquad \wedge\, int\text{-}starts(y_1, x)] \end{array}$$

$$\begin{array}{l} Hath(S, N, u, x) \\ \quad \supset\ (\exists\, y_2)[member(y_2, S) \\ \qquad \wedge\, int\text{-}finishes(y_2, x)] \end{array}$$

Except for the first and last elements of S, every element of S has an element that precedes and follows it:

$$\begin{array}{l} Hath(S, N, u, x) \\ \quad \supset\ (\forall\, y_1)[member(y_1, S) \\ \qquad \supset\ [int\text{-}finishes(y_1, x) \\ \qquad\quad \vee\, (\exists\, y_2)[member(y_2, x) \\ \qquad\qquad \wedge\, int\text{-}meets(y_1, y_2)]]] \end{array}$$

$$\begin{array}{l} Hath(S, N, u, x) \\ \quad \supset\ (\forall\, y_2)[member(y_2, S) \\ \qquad \supset\ [int\text{-}starts(y_2, x) \\ \qquad\quad \vee\, (\exists\, y_1)[member(y_1, x) \\ \qquad\qquad \wedge\, int\text{-}meets(y_1, y_2)]]] \end{array}$$

If time is linearly ordered, the existential quantifier $\exists$ in the last four axioms can be replaced by $\exists\,!$.

Finally, we would like to say that the set S covers x. A simple way to say this is as follows:

$$\begin{array}{l} Hath(S, N, u, x) \\ \quad \supset\ (\forall\, t)[inside(t, x) \\ \qquad \supset\ (\exists\, y)[member(y, S) \\ \qquad\quad \wedge\, in\text{-}interval(t, y)]] \end{array}$$

That is, if an instant t is inside x, there is a smaller unit y that t is inside or the start of.

However, this is a good place to introduce notions of granularity. In describing the temporal properties of some class of events, it may make sense to specify their time with respect to some temporal unit but not with respect to a smaller temporal unit. For example, one might want to talk about an election as a point-like event being at some instant, and specifying the day that instant is in, but not specifying the hour or minute.

To accomodate this, the above axiom can be loosened by applying it only when the instant t is located in *some interval* of size u. The axiom above would be modified as follows:

$$\begin{array}{l} Hath(S,N,u,x) \\ \quad \supset (\forall\, t, y_1)[inside(t,x) \wedge inside(t,y_1) \\ \qquad \wedge\, duration(y_1,u) \\ \qquad \supset (\exists\, y)[member(y,S) \\ \qquad\quad \wedge\, in\text{-}interval(t,y)]] \end{array}$$

Essentially, the conjuncts $inside(t,y_1) \wedge duration(y_1,u)$ specify that t can be viewed at a granularity of u.

This treatment of $Hath$ could be extended to measurable quantities in general.

3.3. The Structure of Temporal Units

We now define predicates true of intervals that are one temporal unit long. For example, $week$ is a predicate true of intervals whose duration is one week.

$$\begin{array}{l} second(T) \equiv seconds(T) = 1 \\ minute(T) \equiv minutes(T) = 1 \\ hour(T) \equiv hours(T) = 1 \\ day(T) \equiv days(T) = 1 \\ week(T) \equiv weeks(T) = 1 \\ month(T) \equiv months(T) = 1 \\ year(T) \equiv years(T) = 1 \end{array}$$

We are now in a position to state the relations between successive temporal units.

$$\begin{array}{l} minute(T) \supset (\exists\, S)Hath(S,60,*Second*,T) \\ hour(T) \supset (\exists\, S)Hath(S,60,*Minute*,T) \\ day(T) \supset (\exists\, S)Hath(S,24,*Hour*,T) \\ week(T) \supset (\exists\, S)Hath(S,7,*Day*,T) \\ year(T) \supset (\exists\, S)Hath(S,12,*Month*,T) \end{array}$$

The relations between months and days are dealt with in Section 4.4.

4. Clock and Calendar

4.1. Time Zones

What hour of the day an instant is in is relative to the time zone. This is also true of minutes, since there are regions in the world, e.g., central Australia, where the hours are not aligned with GMT hours, but are, e.g., offset half an hour. Probably seconds are not relative to the time zone.

Days, weeks, months and years are also relative to the time zone, since, e.g., 2002 began in the Eastern Standard time zone three hours before it began in the Pacific Standard time zone. Thus, predications about all clock and calendar intervals except seconds are relative to a time zone.

This can be carried to what seems like a ridiculous extreme, but turns out to yield a very concise treatment. The Common Era (C.E. or A.D.) is also relative to a time zone, since 2002 years ago, it began three hours earlier in what is now the Eastern Standard time zone than in what is now the Pacific Standard time zone. What we think of as the Common Era is in fact 24 (or more) slightly displaced half-infinite intervals. (We leave B.C.E. to specialized ontologies.)

The principal functions and predicates will specify a clock or calendar unit interval to be the nth such unit in a larger interval. The time zone need not be specified in this predication if it is already built into the nature of the larger interval. That means that the time zone only needs to be specified in the largest interval, that is, the Common Era; that time zone will be inherited by all smaller intervals. Thus, the Common Era can be considered as a function from time zones to intervals.

$$CE(z) = T$$

Fortunately, this counterintuitive conceptualization will usually be invisible and, for example, will not be evident in the most useful expressions for time, in Section 4.5 below. In fact, the CE predication functions as a good place to hide considerations of time zone when they are not relevant.

Time zones should not be thought of as geographical regions. Most places change their time zone twice a year, and a state or county might decide to change its time zone, e.g., from Central Standard to Eastern Standard. Rather it is better to have a separate ontology articulate the relation between geographical regions X times and time zones. For example, it would state that on a certain day and time a particular region changes its time zone from Eastern Standard to Eastern Daylight.

Moreover, time zones that seem equivalent, like Eastern Standard and Central Daylight, should be thought of as separate entities. Whereas they function the same in the time ontology, they do not function the same in the ontology that articulates time and geography. For example, parts of Indiana are always on Eastern Standard Time, and it would be false to say that they shift in April from that to Central Daylight time.

In this treatment it will be assumed there is a set of entities called time zones. Some relations among time zones are discussed in Section 4.5.

4.2. Clock and Calendar Units

The aim of this section is to explicate the various standard clock and calendar intervals. A day as a calender interval begins at and includes midnight and goes until but does not include the next midnight. By contrast, a day as a duration is any interval that is 24 hours in length. The day as a duration was dealt with in Section 3. This section deals with the day as a calendar interval.

It is useful to have three ways of saying the same thing: the clock or calendar interval y is the nth clock or calendar interval of type u in a larger interval x in time zone z. This can be expressed as follows for minutes:

$$min(y, n, x)$$

Because y is uniquely determined by n and x, it can also be expressed as follows:

$$minFn(n, x) = y$$

For stating general properties about clock intervals, it is useful also to have the following way to express the same thing:

$$clock\text{-}int(y, n, u, x)$$

This expression says that y is the nth clock interval of type u in x. For example, the proposition $clock\text{-}int(10:03, 3, *Minute*, [10:00, 11:00])$ holds.

Here u is a member of the set of clock units, that is, one of *Second*, *Minute*, or *Hour*.

In addition, there is a calendar unit function with similar structure:

$$cal\text{-}int(y, n, u, x)$$

This says that y is the nth calendar interval of type u in x. For example, the proposition $cal\text{-}int(12Mar2002, 12, *Day*, Mar2002)$ holds. Here u is one of the calendar units *Day*, *Week*, *Month*, and *Year*.

The unit *DayOfWeek* will be introduced below in Section 4.3.

The relations among these modes of expression are as follows:

$$\begin{aligned}
&sec(y, n, x) \equiv secFn(n, x) = y \\
&\quad \equiv clock\text{-}int(y, n, *sec*, x) \\
&min(y, n, x) \equiv minFn(n, x) = y \\
&\quad \equiv clock\text{-}int(y, n, *min*, x) \\
&hr(y, n, x) \equiv hrFn(n, x) = y \\
&\quad \equiv clock\text{-}int(y, n, *hr*, x) \\
&da(y, n, x) \equiv daFn(n, x) = y \\
&\quad \equiv cal\text{-}int(y, n, *da*, x) \\
&mon(y, n, x) \equiv monFn(n, x) = y \\
&\quad \equiv cal\text{-}int(y, n, *mon*, x) \\
&yr(y, n, x) \equiv yrFn(n, x) = y \\
&\quad \equiv cal\text{-}int(y, n, *yr*, x)
\end{aligned}$$

Weeks and months are dealt with separately below.

The am/pm designation of hours is represented by the function $hr12$.

$$\begin{aligned}
&hr12(y, n, *am*, x) \equiv hr(y, n, x) \\
&hr12(y, n, *pm*, x) \equiv hr(y, n+12, x)
\end{aligned}$$

Each of the calendar intervals is that unit long; a calendar year is a year long.

$$\begin{aligned}
&sec(y, n, x) \supset second(y) \\
&min(y, n, x) \supset minute(y) \\
&hr(y, n, x) \supset hour(y) \\
&da(y, n, x) \supset day(y) \\
&mon(y, n, x) \supset month(y) \\
&yr(y, n, x) \supset year(y)
\end{aligned}$$

A distinction is made above between clocks and calendars because they differ in how they number their unit intervals. The first minute of an hour is labelled with 0; for example, the first minute of the hour [10:00,11:00] is 10:00. The first day of a month is labelled with 1; the first day of March is March 1. We number minutes for the number just completed; we number days for the day we are working on. Thus, if the larger unit has N smaller units, the argument n in clock$-$int runs from 0 to $N-1$, whereas in $cal\text{-}int$ n runs from 1 to N. To state properties true of both clock and calendar intervals, we can use the predicate $cal\text{-}int$ and relate the two notions with the axiom

$$cal\text{-}int(y, n, u, x) \equiv clock\text{-}int(y, n-1, u, x)$$

The type constraints on the arguments of $cal\text{-}int$ are as follows:

$$\begin{aligned}
&cal\text{-}int(y, n, u, x) \\
&\quad \supset interval(y) \wedge integer(n) \\
&\quad\quad \wedge temporal\text{-}unit(u) \wedge interval(x)
\end{aligned}$$

There are properties relating to the labelling of clock and calendar intervals. If N u's hath x and y is the nth u in x, then n is between 1 and N.

$$\begin{aligned}
&cal\text{-}int(y, n, u, x) \wedge Hath(S, N, u, x) \\
&\quad\quad \wedge member(y, S) \\
&\quad \supset 0 < n <= N
\end{aligned}$$

There is a 1st small interval, and it starts the large interval.

$$\begin{aligned}
&Hath(S, N, u, x) \\
&\quad \supset (\exists\, y)[member(y, S) \wedge cal\text{-}int(y, 1, u, x)] \\
&Hath(S, N, u, x) \wedge cal\text{-}int(y, 1, u, x) \\
&\quad \supset int\text{-}starts(y, x)
\end{aligned}$$

There is an nth small interval, and it finishes the large interval.

$$\begin{aligned}
&Hath(S, N, u, x) \\
&\quad \supset (\exists\, y)[member(y, S) \\
&\quad\quad \wedge cal\text{-}int(y, N, u, x)] \\
&Hath(S, N, u, x) \wedge cal\text{-}int(y, N, u, x) \\
&\quad \supset int\text{-}finishes(y, x)
\end{aligned}$$

All but the last small interval have a small interval that succeeds and is met by it.

$$\begin{aligned}
&cal\text{-}int(y1, n, u, x) \wedge Hath(S, N, u, x) \\
&\quad\quad \wedge member(y_1, S) \wedge n < N \\
&\quad \supset (\exists\, y_2)[cal\text{-}int(y_2, n+1, u, x) \\
&\quad\quad \wedge int\text{-}meets(y_1, y_2)]
\end{aligned}$$

All but the first small interval have a small interval that precedes and meets it.

$$\begin{aligned}
&cal\text{-}int(y_2, n, u, x \wedge Hath(S, N, u, x) \\
&\quad\quad \wedge member(y_2, S) \wedge 1 < n \\
&\quad \supset (\exists\, y_1)[cal\text{-}int(y_1, n-1, u, x) \\
&\quad\quad \wedge int\text{-}meets(y_1, y_2)]
\end{aligned}$$

If time is linearly ordered, the existential quantifier $\exists$ can be replaced by $\exists\,!$ in the above axioms.

4.3. Weeks

A calendar week starts at midnight, Saturday night, and goes to the next midnight, Saturday night. It is independent of months and years. However, we can still talk about the nth week in some larger period of time, e.g., the third week of the month or the fifth week of the semester. So the same three modes of representation are appropriate for weeks as well.

$$wk(y,n,x) \equiv wkFn(n,x) = y$$
$$\equiv cal\text{-}int(y,n,*Week*,x)$$

As it happens, the n and x arguments will often be irrelevant.

A calendar week is one week long.

$$wk(y,n,x) \supset week(y)$$

The day of the week is a temporal unit (*DayOfWeek*) in a larger interval, so the three modes of representation are appropriate here as well.

$$dayofweek(y,n,x)$$
$$\equiv dayofweekFn(n,x) = y$$
$$\equiv cal\text{-}int(y,n,*DayOfWeek*,x)$$

Whereas it makes sense to talk about the nth day in a year or the nth minute in a day or the nth day in a week, it does not really make sense to talk about the nth day-of-the-week in anything other than a week. Thus we can restrict the x argument to be a calendar week.

$$dayofweek(y,n,x) \supset (\exists n_1,x_1)wk(x,n_1,x_1)$$

The days of the week have special names in English.

$$dayofweek(y,1,x) \equiv Sunday(y,x)$$
$$dayofweek(y,2,x) \equiv Monday(y,x)$$
$$dayofweek(y,3,x) \equiv Tuesday(y,x)$$
$$dayofweek(y,4,x) \equiv Wednesday(y,x)$$
$$dayofweek(y,5,x) \equiv Thursday(y,x)$$
$$dayofweek(y,6,x) \equiv Friday(y,x)$$
$$dayofweek(y,7,x) \equiv Saturday(y,x)$$

For example, $Sunday(y,x)$ says that y is the Sunday of week x.

A day of the week is also a day of the month (and vice versa), and thus a day long.

$$(\forall y)[[(\exists n,x)dayofweek(y,n,x)]$$
$$\equiv [(\exists n_1,x_1)da(y,n_1,x_1)]]$$

One correspondance will anchor the cycle of weeks to the rest of the calendar, for example, saying that January 1, 2002 was the Tuesday of some week x.

$$(\forall z)(\exists x)Tuesday(dayFn(1,monFn(1,$$
$$yrFn(2002,CE(z)))),x)$$

We can define weekdays and weekend days as follows:

$$weekday(y,x)$$
$$\equiv [Monday(y,x) \vee Tuesday(y,x)$$
$$\vee Wednesday(y,x) \vee Thursday(y,x)$$
$$\vee Friday(y,x)]$$
$$weekendday(y,x)$$
$$\equiv [Saturday(y,x) \vee Sunday(y,x)]$$

4.4. Months and Years

The months have special names in English.

$$mon(y,1,x) \equiv January(y,x)$$
$$mon(y,2,x) \equiv February(y,x)$$
$$mon(y,3,x) \equiv March(y,x)$$
$$mon(y,4,x) \equiv April(y,x)$$
$$mon(y,5,x) \equiv May(y,x)$$
$$mon(y,6,x) \equiv June(y,x)$$
$$mon(y,7,x) \equiv July(y,x)$$
$$mon(y,8,x) \equiv August(y,x)$$
$$mon(y,9,x) \equiv September(y,x)$$
$$mon(y,10,x) \equiv October(y,x)$$
$$mon(y,11,x) \equiv November(y,x)$$
$$mon(y,12,x) \equiv December(y,x)$$

The number of days in a month have to be spelled out for individual months.

$$January(m,y)$$
$$\supset (\exists S)Hath(S,31,*Day*,m)$$
$$March(m,y) \supset (\exists S)Hath(S,31,*Day*,m)$$
$$April(m,y) \supset (\exists S)Hath(S,30,*Day*,m)$$
$$May(m,y) \supset (\exists S)Hath(S,31,*Day*,m)$$
$$June(m,y) \supset (\exists S)Hath(S,30,*Day*,m)$$
$$July(m,y) \supset (\exists S)Hath(S,31,*Day*,m)$$
$$August(m,y)$$
$$\supset (\exists S)Hath(S,31,*Day*,m)$$
$$September(m,y)$$
$$\supset (\exists S)Hath(S,30,*Day*,m)$$
$$October(m,y)$$
$$\supset (\exists S)Hath(S,31,*Day*,m)$$
$$November(m,y)$$
$$\supset (\exists S)Hath(S,30,*Day*,m)$$
$$December(m,y)$$
$$\supset (\exists S)Hath(S,31,*Day*,m)$$

The definition of a leap year is as follows:

$$(\forall z)[leap\text{-}year(y)$$
$$\equiv (\exists n,x)[year(y,n,(CE(z))$$
$$\wedge [divides(400,n)$$
$$\vee [divides(4,n) \wedge \neg divides(100,n)]]]$$

We leave leap seconds to specialized ontologies.

Now the number of days in February can be specified.

$$February(m,y) \wedge leap\text{-}year(y)$$
$$\supset (\exists S)Hath(S,29,*Day*,m)$$
$$February(m,y) \wedge \neg leap\text{-}year(y)$$
$$\supset (\exists S)Hath(S,28,*Day*,m)$$

A reasonable approach to defining month as a unit of temporal measure would be to specify that the start and end points have to be on the same days of successive months.

$$month(T)$$
$$\equiv (\exists d_1,d_2,n,x,m)$$
$$[in\text{-}interval(start\text{-}of(T),d_1)$$
$$\wedge in\text{-}interval(end\text{-}of(T),d_2)$$
$$\wedge da(d_1,n,monFn(m,x))$$
$$\wedge da(d_2,n,monFn(mod{+}(m,1,12),x))]$$

Here $mod+$ is modulo addition to take care of months spanning December and January. So the month as a measure of duration would be related to days as a measure of duration only indirectly, mediated by the calendar.

To say that July 4 is a holiday in the United States one could write

$$(\forall\, d, m, y)[da(d, 4, m) \wedge July(m, y) \\ \supset holiday(d, USA)]$$

4.5. Time Stamps

Standard notation for times list the year, month, day, hour, minute, and second. It is useful to define a predication for this.

$$time\text{-}of(t, y, m, d, h, n, s, z) \\ \equiv in\text{-}interval(t, secFn(s, minFn(n, hrFn(h, \\ daFn(d, monFn(m, yrFn(y, CE(z))))))))$$

For example, an instant t has the time

5:14:35pm PST, Wednesday, February 6, 2002

if the following properties hold for t:

$$time\text{-}of(t, 2002, 2, 6, 17, 14, 35, *PST*) \\ (\exists\, w, x)[in\text{-}interval(t, w) \\ \wedge Wednesday(w, x)]$$

The second line says that t is in the Wednesday w of some week x.

The relations among time zones can be expressed in terms of the time$-$of predicate. Two examples are as follows:

$$h < 8 \supset [time\text{-}of(t, y, m, d, h, n, s, *GMT*) \\ \equiv time\text{-}of(t, y, m, d-1, h+16, n, s, *PST*)] \\ h \geq 8 \\ \supset [time\text{-}of(t, y, m, d, h, n, s, *GMT*) \\ \equiv time\text{-}of(t, y, m, d, h-8, n, s, *PST*)]$$

$$time\text{-}of(t, y, m, d, h, n, s, *EST*) \\ \equiv time\text{-}of(t, y, m, d, h, n, s, *CDT*)$$

5. Deictic Time

Deictic temporal concepts, such as "now", "today", "tomorrow night", and "last year", are more common in natural language texts than they will be in descriptions of Web resources, and for that reason we are postponing a development of this domain until the first three are in place. But since most of the content on the Web is in natural language, ultimately it will be necessary for this ontology to be developed. It should, as well, mesh well with the annotation standards used in automatic tagging of text.

We expect that the key concept in this area will be a relation now between an instant and an utterance or document.

$$now(t, d)$$

The concept of "today" would also be relative to a document, and would be defined as follows:

$$today(T, d) \\ \equiv (\exists\, t, n, x)[now(t, d) \wedge in\text{-}interval(t, T) \\ \wedge da(T, n, x)]$$

That is, T is today with respect to document d if and only if there is an instant t in T that is now with respect to the document and T is a calendar day (and thus the nth calendar day in some interval x).

Present, past and future can be defined in the obvious way in terms of now and before.

Another feature of a treatment of deictic time would be an axiomatization of the concepts of "last", "this", and "next" on anchored sequences of temporal entities.

6. Aggregates of Temporal Entities

A number of common expressions and commonly used properties are properties of sequences of temporal entities. These properties may be properties of all the elements in the sequence, as in "every Wednesday", or they may be properties of parts of the sequence, as in "three times a week" or "an average of once a year". We are also postponing development of this domain until the first three domains are well in hand.

This may be the proper locus of a duration arithmetic, since we may want to know the total time an intermittant process is in operation.

7. Vague Temporal Concepts

In natural language a very important class of temporal expressions are inherently vague. Included in this category are such terms as "soon", "recently", "late", and "a little while". These require an underlying theory of vagueness, and in any case are probably not immediately critical for the Semantic Web. This area will be postponed for a little while.

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8. References

Allen, James F. and Henry A. Kautz. 1985. "A model of naive temporal reasoning." *Formal Theories of the Commonsense World*, ed. by Jerry R. Hobbs and Robert C. Moore, Ablex Publishing Corp., pp. 251-268.