CSCI 360
Introduction to Artificial Intelligence

Instructor: Wei-Min Shen

Week 4.1
Status Check and Review

• Status check
  – How is your project?
  – Have you done your homework?
  – Why there is no discussion on Piazza?
  – Why there is no one shown up for TA?

• Review of last lecture

• What is today’s lecture?
  – What are the examples?
Constraint Satisfaction (review)

• Examples: Map Coloring, can you think of another?
• How to formalize the problem?
  – *Search*: variables, domains, constrained relations
  – Do the subset independent?
  – Do it contain cycles?
• How to solve the problem? (Randomly or “Best”-First)
  – Global Backtracking
    • Order variables: the most constrained variables first
    • Satisfy one at a time, if stuck, backtracking
  – Local Search: heuristic, find the min-conflicts solution locally
  – Centralized vs. Distributed (DCOP)
• **HW**: Magic square, define it and solve it
This Week’s Lecture (1): Logics

- The real world and its representation by logics
- The world you created in simulation (e.g. wumpus)
  - Syntax (on paper, \(\alpha, \beta\))
  - Semantics (in world/model M), ”satisfy”
- Propositional logic: truth table for
  - Theorem proving, inference rule, resolution, horn clause, forward/backward chaining,
  - Propositional agent: current state, goal state, local search, global search
- HW: PL rules for Wumpus or Tic-Tac-Toe
This Week’s Lecture (2): First-Order Logic

• Syntax and semantics
  – Add variables (predicates) and quantifiers

• Knowledge engineering
  – Capture and represent human knowledge, e.g., using FOL

• Inference in FOL
  – Modus Ponens with variables
    • Unification: finding the values for variables
  – 1. Forward chaining
    • E.g. production systems, ACT, SOAR
  – 2. Backward chaining
    • E.g., Prolog
  – 3. Resolution
    • Convert to CNF, eliminating P, -P.

• HW: FOL rules for Wumpus or Tic-Tac-Toe
Today’s Lecture

• Logic Representation of the world
• Propositional logic
• First-order logic
• FOL Inferences
Key Concepts of Logic

- Syntax
- Semantics
- Entailment
- Inference
- Soundness
- Completeness
- Inference Rules
- Normal Forms
- Truth Tables
- Reasoning
Knowledge and Reasoning

- Knowledge representation
- Logic and representation
- Propositional (Boolean) logic
- Normal forms
- Inference in propositional logic
- Wumpus world example
Symbols vs the Real World

Symbol System

Objects:
+BLOCKA+
+BLOCKB+
+BLOCKC+
......

Relations:
P_1:(IS_ON +BLOCKA+ +BLOCKB+)
P_2:(IS_RED +BLOCKA+)
......

“Knowledge Base”
“Robot Brain”

World
Knowledge-Based Agent

- Agent that uses prior or acquired knowledge to achieve its goals
  - Can make more efficient decisions
  - Can make informed decisions
- Knowledge Base (KB): contains a set of representations of facts about the Agent’s environment
- Each representation is called a sentence
- Use some knowledge representation language, to TELL it what to know e.g., (temperature 72F)
- ASK agent to query what to do
- Agent can use inference to deduce new facts from TELLed facts
Generic knowledge-based agent

function KB-AGENT( percept) returns an action
  static: KB, a knowledge base
          t, a counter, initially 0, indicating time
  TELL(KB, MAKE-PERCEPT-SENTENCE( percept, t))
  action ← ASK(KB, MAKE-ACTION-QUERY(t))
  TELL(KB, MAKE-ACTION-SENTENCE(action, t))
  t ← t + 1
  return action

1. TELL KB what was perceived
   Uses a KRL to insert new sentences, representations of facts, into KB

2. ASK KB what to do.
   Uses logical reasoning to examine actions and select best.
Wumpus world example

Percepts Breeze, Glitter, Smell

Actions Left turn, Right turn, Forward, Grab, Release, Shoot

Goals Get gold back to start without entering pit or wumpus square

Environment

Squares adjacent to wumpus are smelly
Squares adjacent to pit are breezy
Glitter if and only if gold is in the same square
Shooting kills the wumpus if you are facing it
Shooting uses up the only arrow
Grabbing picks up the gold if in the same square
Releasing drops the gold in the same square
Wumpus world characterization

- Deterministic?
- Accessible?
- Static?
- Discrete?
- Episodic?
Wumpus world characterization

- Deterministic? Yes – outcome exactly specified.
- Accessible? No – only local perception.
- Static? Yes – Wumpus and pits do not move.
- Discrete? Yes
- Episodic? (Yes) – because static.
### Example Solution Steps

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### Diagram

- **A** = Agent
- **B** = Breeze
- **G** = Glitter, Gold
- **OK** = Safe square
- **P** = Pit
- **S** = Stench
- **V** = Visited
- **W** = Wumpus

**Grid:**

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**Grid: B in 2,1**

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**Grid: 1,1 V**

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**Grid: Move to 1,2 (only option)**

No perception → 1,2 and 2,1 OK
Move to 2,1

B in 2,1 → 2,2 or 3,1 P?
1,1 V → no P in 1,1
Move to 1,2 (only option)
### Example Solution Steps

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<td>A</td>
<td>2,2</td>
<td>3,2</td>
<td>4,2</td>
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<tr>
<td>1,1</td>
<td>V</td>
<td>2,1</td>
<td>3,1</td>
<td>4,1</td>
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<td>2,2</td>
<td>V</td>
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</table>

- S in 1,2 and No S when in 2,1 → 1,3 or 1,2 has W
- 1,2 OK → 1,3 W
- No B in 1,2 → 2,2 OK & 3,1 P
Exploring a Wumpus world

A = Agent
B = Breeze
S = Smell
P = Pit
W = Wumpus
OK = Safe
V = Visited
G = Glitter
Exploring a Wumpus world

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Exploring a Wumpus world

A = Agent
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W = Wumpus
OK = Safe
V = Visited
G = Glitter
Other Tight Spots

Breeze in (1,2) and (2,1) 
⇒ no safe actions

Assuming pits uniformly distributed, (2,2) is most likely to have a pit

Smell in (1,1) 
⇒ cannot move

Can use a strategy of coercion:
shoot straight ahead
wumpus was there ⇒ dead ⇒ safe
wumpus wasn’t there ⇒ safe
Logic in General

Logics are formal languages for representing information such that conclusions can be drawn.

Syntax defines the sentences in the language.

Semantics define the "meaning" of sentences; i.e., define truth of a sentence in a world.

E.g., the language of arithmetic:

$x + 2 \geq y$ is a sentence; $x^2 + y >$ is not a sentence.

$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number $y$.

$x + 2 \geq y$ is true in a world where $x = 7, \ y = 1$.

$x + 2 \geq y$ is false in a world where $x = 0, \ y = 6$. 
Types of logic

Logics are characterized by what they commit to as “primitives”

**Ontological commitment**: what exists—facts? objects? time? beliefs?

**Epistemological commitment**: what states of knowledge?

<table>
<thead>
<tr>
<th>Language</th>
<th>Ontological Commitment</th>
<th>Epistemological Commitment</th>
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<tbody>
<tr>
<td>Propositional logic</td>
<td>facts</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>First-order logic</td>
<td>facts, objects, relations</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>Temporal logic</td>
<td>facts, objects, relations, times</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>Probability theory</td>
<td>facts</td>
<td>degree of belief 0...1</td>
</tr>
<tr>
<td>Fuzzy logic</td>
<td>degree of truth</td>
<td>degree of belief 0...1</td>
</tr>
</tbody>
</table>
Physical Symbol System

+BLOCKA+
+BLOCKB+
+BLOCKC+

P₁: (IS_ON +BLOCKA+ +BLOCKB+)
P₂: (IS_RED +BLOCKA+)
Truth depends on Interpretation

Representation 1

A

B

ON(A,B) \( T \)

ON(B,A) \( F \)

Representation 2

ON(A,B) \( F \) A

ON(B,A) \( T \) B

World
Entailment

\[ KB \models \alpha \]

Knowledge base \( KB \) entails sentence \( \alpha \)
if and only if
\( \alpha \) is true in all worlds where \( KB \) is true

E.g., the KB containing “the Giants won” and “the Reds won”
entails “Either the Giants won or the Reds won”
Logic as a Representation of a World

Representation: Sentences \(\text{entails}\) Sentence

Refers to (Semantics)

World \(\text{follows}\) Facts

Facts
Models ("Possible Worlds")

Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated.

We say \( m \) is a model of a sentence \( \alpha \) if \( \alpha \) is true in \( m \).

\( M(\alpha) \) is the set of all models of \( \alpha \).

Then \( KB \models \alpha \) if and only if \( M(KB) \subseteq M(\alpha) \).

E.g., \( KB = \) Giants won and Reds won

\( \alpha = \) Giants won
Inference

\[ KB \vdash_i \alpha = \text{sentence } \alpha \text{ can be derived from } KB \text{ by procedure } i \]

**Soundness:** \( i \) is sound if

whenever \( KB \vdash_i \alpha \), it is also true that \( KB \models \alpha \)

**Completeness:** \( i \) is complete if

whenever \( KB \models \alpha \), it is also true that \( KB \vdash_i \alpha \)

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the \( KB \).
“Entail” $\equiv$ “Inference”

KB inference $\alpha$: derived by a procedure

KB entails $\alpha$

Model of KB is a subset of Model of $\alpha$

Model of Symbols: $M(KB), M(\alpha), \ldots$

“possible worlds”?

In the Real World:
Basic Symbols

- Expressions only evaluate to either “true” or “false”

- \( P \) “\( P \) is true”
- \( \neg P \) “\( P \) is false”
- \( P \lor Q \) “either \( P \) is true or \( Q \) is true or both”
- \( P \land Q \) “both \( P \) and \( Q \) are true”
- \( P \rightarrow Q \) “if \( P \) is true, then \( Q \) is true”
- \( P \iff Q \) “\( P \) and \( Q \) are either both true or both false”

**Operations**
- negation
- disjunction
- conjunction
- implication
- equivalence
Propositional Logic: Syntax

Propositional logic is the simplest logic—illustrates basic logical operations.

- The proposition symbols $P_1$, $P_2$ etc are sentences.
- If $S$ is a sentence, $\neg S$ is a sentence.
- If $S_1$ and $S_2$ is a sentence, $S_1 \land S_2$ is a sentence.
- If $S_1$ and $S_2$ is a sentence, $S_1 \lor S_2$ is a sentence.
- If $S_1$ and $S_2$ is a sentence, $S_1 \Rightarrow S_2$ is a sentence.
- If $S_1$ and $S_2$ is a sentence, $S_1 \Leftrightarrow S_2$ is a sentence.
Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.  \( A \quad B \quad C \)

    \( \text{True} \quad \text{True} \quad \text{False} \)

Rules for evaluating truth with respect to a model \( m \):

\[
\begin{align*}
\neg S & \text{ is true iff } S \text{ is false} \\
S_1 \land S_2 & \text{ is true iff } S_1 \text{ is true and } S_2 \text{ is true} \\
S_1 \lor S_2 & \text{ is true iff } S_1 \text{ is true or } S_2 \text{ is true} \\
S_1 \Rightarrow S_2 & \text{ is true iff } S_1 \text{ is false or } S_2 \text{ is true} \\
\text{i.e., is false iff } S_1 & \text{ is true and } S_2 \text{ is false} \\
S_1 \Leftrightarrow S_2 & \text{ is true iff } S_1 \Rightarrow S_2 \text{ is true and } S_2 \Rightarrow S_1 \text{ is true}
\end{align*}
\]
Truth Tables

• Truth value: whether a statement is true or false.
• Truth table: complete list of truth values for a statement given all possible values of the individual atomic expressions.

Example:

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P ∨ Q</th>
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</table>
Truth Tables for Basic Connectives

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>\neg P</th>
<th>\neg Q</th>
<th>P \lor Q</th>
<th>P \land Q</th>
<th>P \rightarrow Q</th>
<th>P \leftrightarrow Q</th>
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Propositional Logic: Basic Manipulation Rules

- \( \neg(\neg A) = A \)  
  Double negation

- \( \neg(A \land B) = (\neg A) \lor (\neg B) \)  
  Negated “and”

- \( \neg(A \lor B) = (\neg A) \land (\neg B) \)  
  Negated “or”

- \( A \land (B \lor C) = (A \land B) \lor (A \land C) \)  
  Distributivity of \( \land \) on \( \lor \)

- \( A \lor (B \land C) = (A \lor B) \land (A \lor C) \)  
  Distributivity of \( \lor \) on \( \land \)

- \( A \Rightarrow B = (\neg A) \lor B \)  
  by definition

- \( \neg(A \Rightarrow B) = A \land (\neg B) \)  
  using negated or

- \( A \Leftrightarrow B = (A \Rightarrow B) \land (B \Rightarrow A) \)  
  by definition

- \( \neg(A \Leftrightarrow B) = (A \land (\neg B)) \lor (B \land (\neg A)) \)  
  using negated and \& or

- ...

40
Propositional Inference: Enumeration Method

Let \( \alpha = A \lor B \) and \( KB = (A \lor C) \land (B \lor \neg C) \)

Is it the case that \( KB \models \alpha \)?
Check all possible models—\( \alpha \) must be true wherever \( KB \) is true

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Enumeration: Solution

KB |= alpha?

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<td>α</td>
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Propositional Inference: Normal Forms

Other approaches to inference use syntactic operations on sentences, often expressed in standardized forms

**Conjunctive Normal Form (CNF—universal)**

A *conjunction of disjunctions of literals* clauses

E.g., \((A \lor \neg B) \land (B \lor \neg C \lor \neg D)\)

**Disjunctive Normal Form (DNF—universal)**

A *disjunction of conjunctions of literals* terms

E.g., \((A \land B) \lor (A \land \neg C) \lor (A \land \neg D) \lor (\neg B \land \neg C) \lor (\neg B \land \neg D)\)

**Horn Form (restricted)**

A *conjunction of Horn clauses* (clauses with \(\leq 1\) positive literal)

E.g., \((A \lor \neg B) \land (B \lor \neg C \lor \neg D)\)

Often written as set of implications:

\(B \Rightarrow A\) and \((C \land D) \Rightarrow B\)
Deriving Expressions from Functions

• Given a boolean function in truth table form, find a propositional logic expression for it that uses only \( V, \wedge \) and \( \neg \).

• **Idea:** Easily do it by disjoining the “T” rows of the truth table.

Example: XOR function

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>RESULT</th>
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<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
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RESULT = \((P \wedge (\neg Q)) \lor ((\neg P) \wedge Q)\)
A More Formal Approach

• How To construct a logical expression in disjunctive normal form from a truth table?

- Build a “minterm” for each row of the table, where:
  - For each variable whose value is T in that row, include the variable in the minterm
  - For each variable whose value is F in that row, include the negation of the variable in the minterm
  - Link variables in minterm by conjunctions

- The expression consists of the **disjunction of all minterms**
Example: Adder with Carry

Takes 3 variables in: x, y and ci (carry-in); yields 2 results: sum (s) and carry-out (co).
We use other notations, here we assume T = 1, F = 0, V = OR, ^ = AND, ¬ = NOT.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>ci</th>
<th>co</th>
<th>s</th>
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<tbody>
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The logical expression for co is:

\[(\text{NOT } x \text{ AND } y \text{ AND } ci) \text{ OR } (x \text{ AND } \text{NOT } y \text{ AND } ci) \text{ OR } (x \text{ AND } y \text{ AND } \text{NOT } ci) \text{ OR } (x \text{ AND } y \text{ AND } ci)\]

The logical expression for s is:

\[(\text{NOT } x \text{ AND } \text{NOT } y \text{ AND } ci) \text{ OR } (\text{NOT } x \text{ AND } y \text{ AND } \text{NOT } ci) \text{ OR } (x \text{ AND } \text{NOT } y \text{ AND } \text{NOT } ci) \text{ OR } (x \text{ AND } y \text{ AND } ci)\]
Tautologies

• Logical expressions that are always true. Can be simplified.

Examples:

T
T V A
A V (¬A)
¬(A ^ (¬A))
A ↔ A
((P V Q) ↔ P) V (¬P ^ Q)
(P ↔ Q) ⇒ (P ⇒ Q)
Validity and Satisfiability

A sentence is valid if it is true in all models
  e.g., $A \lor \neg A$, $A \Rightarrow A$, $(A \land (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem
  $KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model
  e.g., $A \lor B$, $C$

A sentence is unsatisfiable if it is true in no models
  e.g., $A \land \neg A$

Satisfiability is connected to inference via the following:
  $KB \models \alpha$ if and only if $(KB \land \neg \alpha)$ is unsatisfiable
  i.e., prove $\alpha$ by reductio ad absurdum
Logic Proof Methods

Proof methods divide into (roughly) two kinds:

Model checking
- truth table enumeration (sound and complete for propositional)
- heuristic search in model space (sound but incomplete)
  e.g., the GSAT algorithm (Ex. 5.15)

Application of inference rules
- Legitimate (sound) generation of new sentences from old
  **Proof** = a sequence of inference rule applications
    - Can use inference rules as operators in a standard search alg.
Inference Rules

◊ **Modus Ponens** or **Implication-Elimination**: (From an implication and the premise of the implication, you can infer the conclusion.)

\[
\alpha \implies \beta, \quad \alpha \\
\hline
\beta
\]

◊ **And-Elimination**: (From a conjunction, you can infer any of the conjuncts.)

\[
\alpha_1 \land \alpha_2 \land \ldots \land \alpha_n \\
\hline
\alpha_i
\]

◊ **And-Introduction**: (From a list of sentences, you can infer their conjunction.)

\[
\alpha_1, \alpha_2, \ldots, \alpha_n \\
\hline
\alpha_1 \land \alpha_2 \land \ldots \land \alpha_n
\]

◊ **Or-Introduction**: (From a sentence, you can infer its disjunction with anything else at all.)

\[
\alpha_i \\
\hline
\alpha_1 \lor \alpha_2 \lor \ldots \lor \alpha_n
\]
**Inference Rules (cont.)**

◊ **Double-Negation Elimination:** (From a doubly negated sentence, you can infer a positive sentence.)

\[
\begin{align*}
\neg\neg\alpha & \\
\hline
\alpha
\end{align*}
\]

◊ **Unit Resolution:** (From a disjunction, if one of the disjuncts is false, then you can infer the other one is true.)

\[
\begin{align*}
\alpha \vee \beta, \quad \neg\beta & \\
\hline
\alpha
\end{align*}
\]

◊ **Resolution:** (This is the most difficult. Because \(\beta\) cannot be both true and false, one of the other disjuncts must be true in one of the premises. Or equivalently, implication is transitive.)

\[
\begin{align*}
\alpha \vee \beta, \quad \neg\beta \vee \gamma & \\
\hline
\alpha \vee \gamma
\end{align*}
\]

or equivalently

\[
\begin{align*}
\neg\alpha \Rightarrow \beta, \quad \beta \Rightarrow \gamma & \\
\hline
\neg\alpha \Rightarrow \gamma
\end{align*}
\]
Wumpus World: example

• **Facts:** Percepts inject (TELL) facts into the KB
  – [Stench at 1,1 and 2,1] → S(1,1); S(2,1)

• **Rules:** if square has no stench then neither the square or adjacent square contain the Wumpus
  – R1: ~S(1,1) → ~W(1,1) ^ ~W(1,2) ^ ~W(2,1)
  – R2: ~S(2,1) → ~W(1,1) ^ ~W(2,1) ^ ~W(2,2) ^ ~W(3,1)
  – ...

• **Inference:**
  – KB contains ~S(1,1) then using Modus Ponens we can infer 
    ~W(1,1) ^ ~W(1,2) ^ ~W(2,1)
  – Using And-Elimination we can get: ~W(1,1); ~W(1,2); ~W(2,1)
  – ...

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Limitations of Propositional Logic

1. It has very limited expressiveness (no variables)
   - Each rule has to be represented for each situation:
     e.g., “don’t go forward if the wumpus is in front of you” would take 64 rules

2. It cannot keep track of changes
   - If one needs to track changes, e.g., where the agent has been before then we need a timed-version of each rule.
     To track 100 steps we’ll then need 6400 rules for the previous example.

It’s hard to write and maintain such a huge rule-base
Inference becomes intractable
Logical agents apply inference to a knowledge base to derive new information and make decisions.

Basic concepts of logic:
- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundness: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Propositional logic suffices for some of these tasks.

Truth table method is sound and complete for propositional logic.
Logic and Reasoning

• Knowledge Base (KB): contains a set of sentences expressed using a knowledge representation language
  • TELL: operator to add a sentence to the KB
  • ASK: to query the KB
• Logics are KRLs where conclusions can be drawn
  • Syntax
  • Semantics
• Entailment: KB |= a iff a is true in all worlds where KB is true
• Inference: KB |–i a = sentence a can be derived from KB using procedure i
  • Sound: whenever KB |–i a, then KB |= a is true
  • Complete: whenever KB |= a, then KB |–i a
Propositional Logic Syntax

Propositional logic is the simplest logic—illustrates basic logical connectives.

The proposition symbols $P_1$, $P_2$ etc are sentences.

If $S$ is a sentence, $\neg S$ is a sentence.

If $S_1$ and $S_2$ is a sentence, $S_1 \land S_2$ is a sentence.

If $S_1$ and $S_2$ is a sentence, $S_1 \lor S_2$ is a sentence.

If $S_1$ and $S_2$ is a sentence, $S_1 \Rightarrow S_2$ is a sentence.

If $S_1$ and $S_2$ is a sentence, $S_1 \Leftrightarrow S_2$ is a sentence.
Propositional Logic Semantics

Each model specifies true/false for each proposition symbol

E.g.  \( A \quad B \quad C \)

\[ \text{True} \quad \text{True} \quad \text{False} \]

Rules for evaluating truth with respect to a model \( m \):

- \( \neg S \) is true iff \( S \) is false
- \( S_1 \land S_2 \) is true iff \( S_1 \) is true and \( S_2 \) is true
- \( S_1 \lor S_2 \) is true iff \( S_1 \) is true or \( S_2 \) is true
- \( S_1 \Rightarrow S_2 \) is true iff \( S_1 \) is false or \( S_2 \) is true
  i.e., is false iff \( S_1 \) is true and \( S_2 \) is false
- \( S_1 \Leftrightarrow S_2 \) is true iff \( S_1 \Rightarrow S_2 \) is true and \( S_2 \Rightarrow S_1 \) is true
Propositional Logic Inference Rules

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  \[
  \alpha \Rightarrow \beta, \quad \alpha \quad \quad \Rightarrow \quad \quad \beta
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  \]

- **Resolution**: (This is the most difficult. Because \(\beta\) cannot be both true and false, one of the other disjuncts must be true in one of the premises. Or equivalently, implication is transitive.)
  \[
  \alpha \lor \beta, \quad \neg \beta \lor \gamma \quad \text{or equivalently} \quad \neg \alpha \Rightarrow \beta, \quad \beta \Rightarrow \gamma
  \]

\[\alpha \lor \gamma\]