ADIABATIC PREPARATION OF ENCODED QUANTUM STATES

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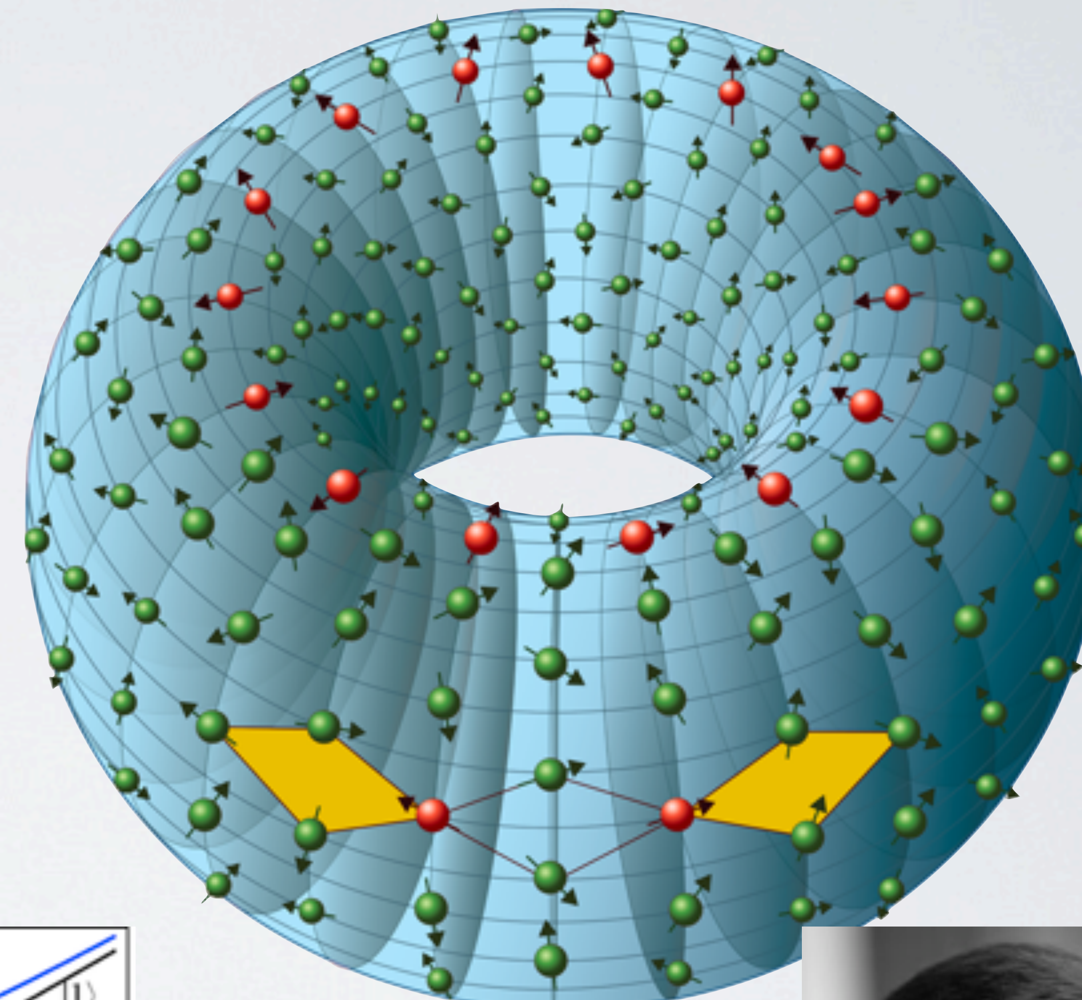
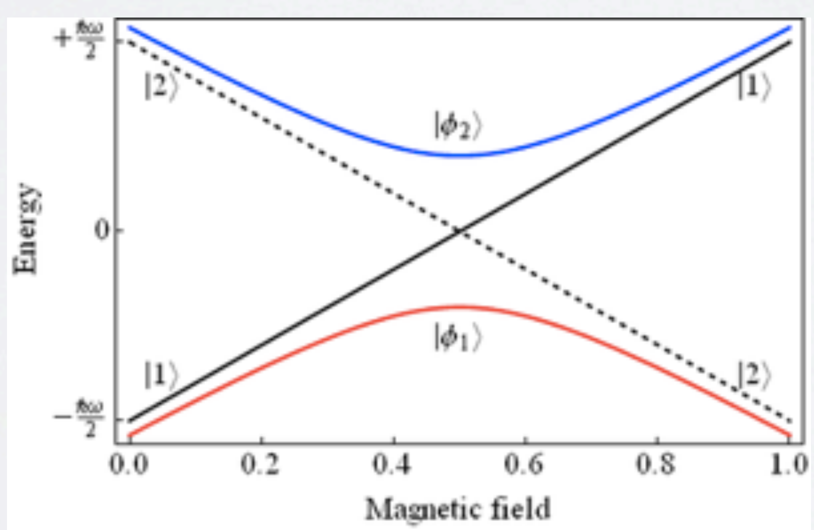
QUANTUM NOISE

- Storing single observable not enough.
- Decoherence deteriorates "quantum" observables.
- Can we store quantum information?

KITAEV'S IDEA

- Locally defined quantum error correcting code (QECC).
- Hamiltonian with degenerate ground space (GS).
- Constructed examples of "topologically protected" GS.

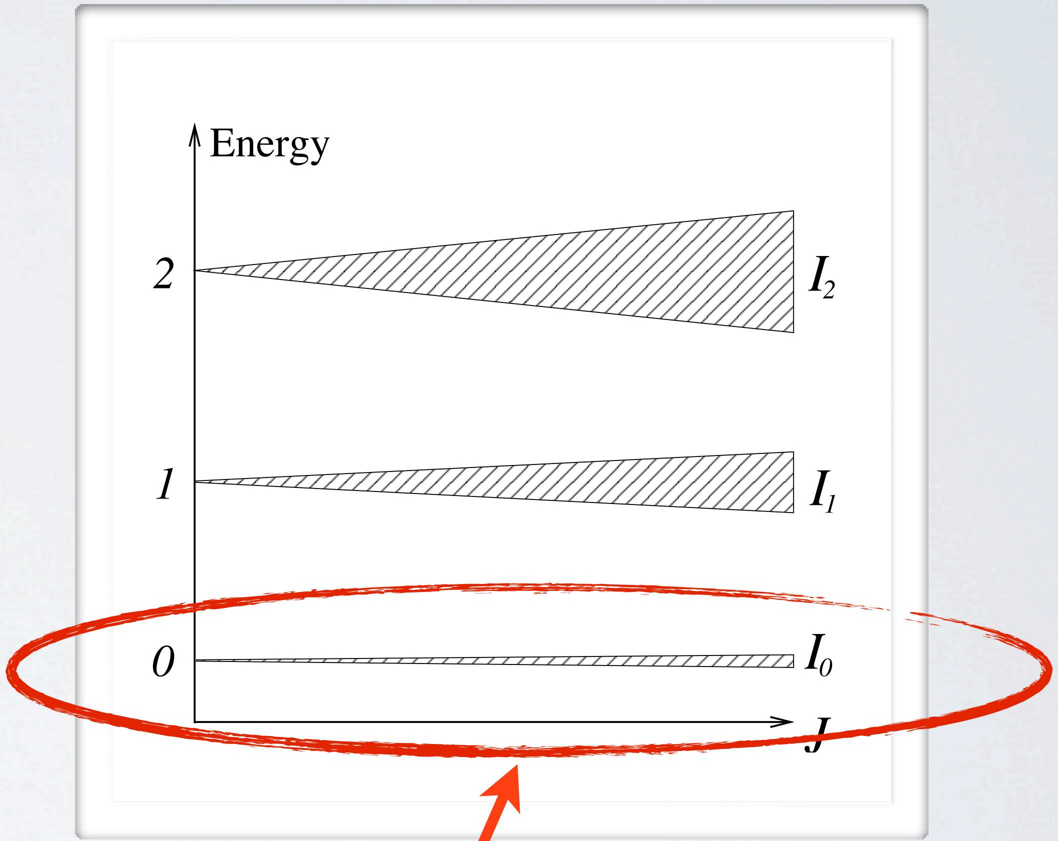




Kitaev, A.Y. (2003). Fault-tolerant quantum computation by anyons. Annals of Physics, 303(1), 2–30.

ROBUST ENERGY DEGENERACY

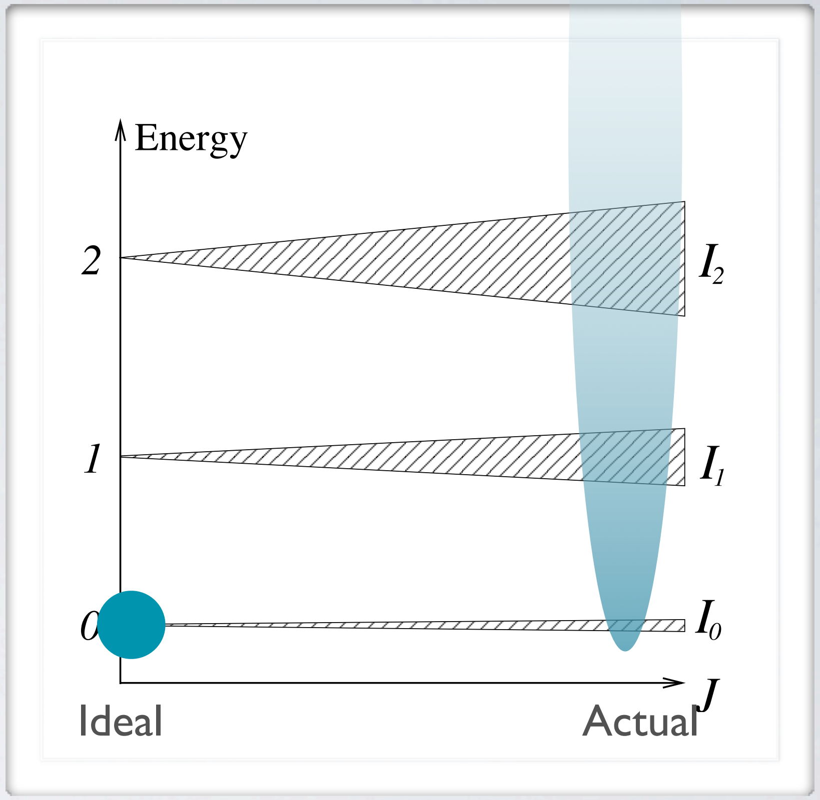
- Quantum information storage.
- Decoherence = uncontrolled phase = uncontrolled energy or time
- Use robust "zero-modes" !



Code space = ground space

Bravyi, S., Hastings, M. B., & Michalakis, S. (2010). Topological quantum order: Stability under local perturbations. Journal of Mathematical Physics, 51(9), 093512. doi:10.1063/1.3490195

ACTUAL VS. IDEAS GROUND SPACE



$$\tilde{P} = U P_0 U^\dagger \qquad P_0 = U^\dagger \tilde{P} U$$

- Degeneracy is robust. code space is not!

$$\tilde{H} = U H_0 U^\dagger = H_0 + (U - 1) H_0 U^\dagger + H_0 (U^\dagger - 1)$$

$$U = \prod_s U_j = \prod_j \exp(\varepsilon T_j) \qquad tr[P_0 \tilde{P}] \propto tr[P_0] \exp(-\varepsilon N)$$

- Ideal code space has large contribution of perturbed excitations.

- Excitations introduce errors in t < O(log(L)) for the toric code.

Pastawski, F., Kay, A., Schuch, N., & Cirac, I. (2010). Limitations of Passive Protection of Quantum Information. Quantum Information and Computation, 10(7&8), 0580–0618.

ADIABATIC PREPARATION OF ENCODED QUANTUM STATES

Prepares **actual GS** by design.

CAN IT BE DONE?

(The gap closes)

IS IT ROBUST?

CAN WE ENCODE INFORMATION?

OUTLINE

- Toric code Hamiltonian
  - Adiabatic interpolation.
  - Joint symmetries.
  - Phase transitions & boundaries.
- Supercoherent qubit.
- Color code (Steane 7 qubit)Accessible states.
- Conclusions

TORIC CODE & HAMILTONIAN

Plaquete and vertex **anyons**

$$P_p = \left(\mathbb{1} + \bigotimes_{e \text{ of } p} Z_e \right) /2$$

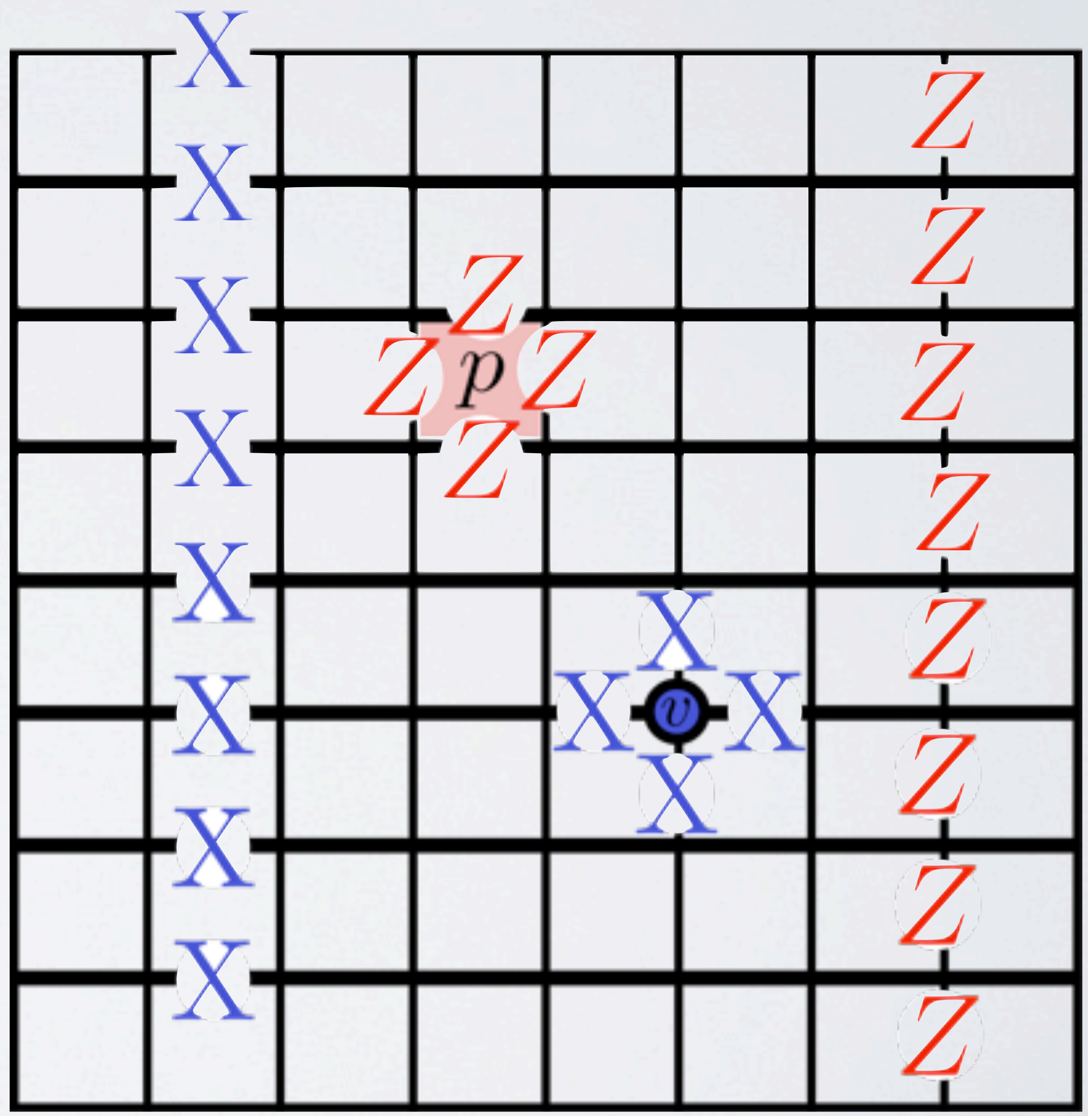
$$P_v = \left(\mathbb{1} + \bigotimes_{e \text{ at } v} X_e \right) /2$$

Ground space projector

$$P_0 = \prod_p P_p \prod_v P_v$$

$$H_{TC} = -\sum_v P_v - \sum_p P_p$$

Parameters: $[[2L^2, 2, L]]$



Kitaev, A.Y. (2003). Fault-tolerant quantum computation by anyons. Annals of Physics, 303(1), 2–30.

ADIABATIC PREPARATION OF TOPOLOGICAL ORDER

Hamma, A., & Lidar, D. A. (2008)

$$H(s) = H_U + (1-s)H_\xi + sH_g \qquad U \gg g, \xi$$

$$H_U = -U\sum_p P_p \qquad H_\xi = \xi \sum_e Z_e \qquad H_g = -g\sum_v P_v$$

- No plaquete anyons: H_U is a joint symmetry of the system.
- Lattice gauge duality mapping gives gap: $\Delta(L) \approx O(L^{-1})$
- Sectors are topologically protected from perturbations.

ADIABATIC INTERPOLATION

- From uniform field to toric code

$$H(s) = (1-s)H_P + sH_{TC}$$

Magnon quasiparticles

$$H_P = -\vec{h} \cdot \sum_e \vec{\sigma}_e$$

Unique GS.

Constant gap

$$\vec{\sigma} = \{X, Y, Z\}$$

Anyon quasiparticles

$$H_{TC} = -\sum_v S_v - \sum_p S_p$$

4-fold degenerate GS

Topological gap

JOINT SYMMETRIES & CONSERVED QUANTITIES

No plaquete anyons

$$\vec{h}\cdot\vec{\sigma} = \pm Z \quad \Rightarrow \quad \forall p, s : [S_p, H(s)] = 0$$

$$\forall s, q \in \{1,2\} : [\bar{Z}_q, H(s)] = 0$$

No vertex anyons

$$\vec{h}\cdot\vec{\sigma} = \pm X \quad \Rightarrow \quad \forall v, s : [S_v, H(s)] = 0$$

$$\forall s, q \in \{1,2\} : [\bar{X}_q, H(s)] = 0$$

Less conserved quantities in Y direction.

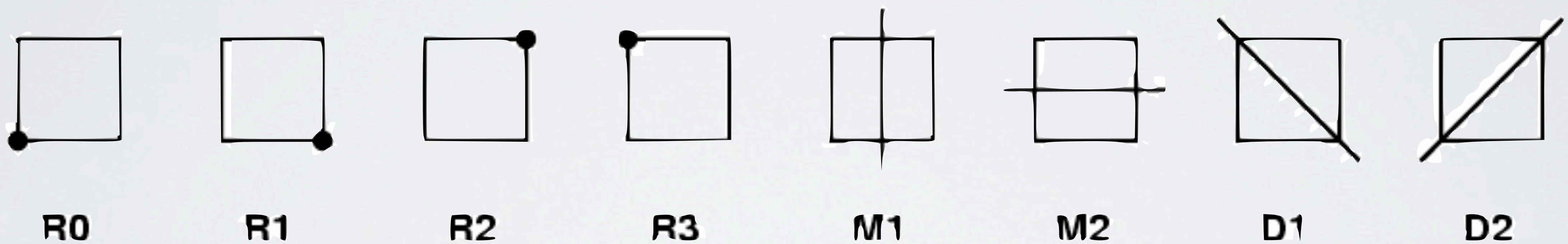
$$\vec{h}\cdot\vec{\sigma} = \pm Y \quad \Rightarrow \quad \forall s, q \in \{1,2\} : [\bar{Y}_q, H(s)] = 0$$

Hamma, A., & Lidar, D. A. (2008). Adiabatic Preparation of Topological Order. Physical Review Letters, 100(3), 030502–4.

OTHER JOINT SYMMETRIES

Translational invariance: $\forall s, \hat{\imath} : [T_{\hat{\imath}}, H(s)] = 0$

D4 symmetry on square: $\forall s, R \in D4 : [R, H(s)] = 0$



Unique initial GS shares symmetries (+1 eig).

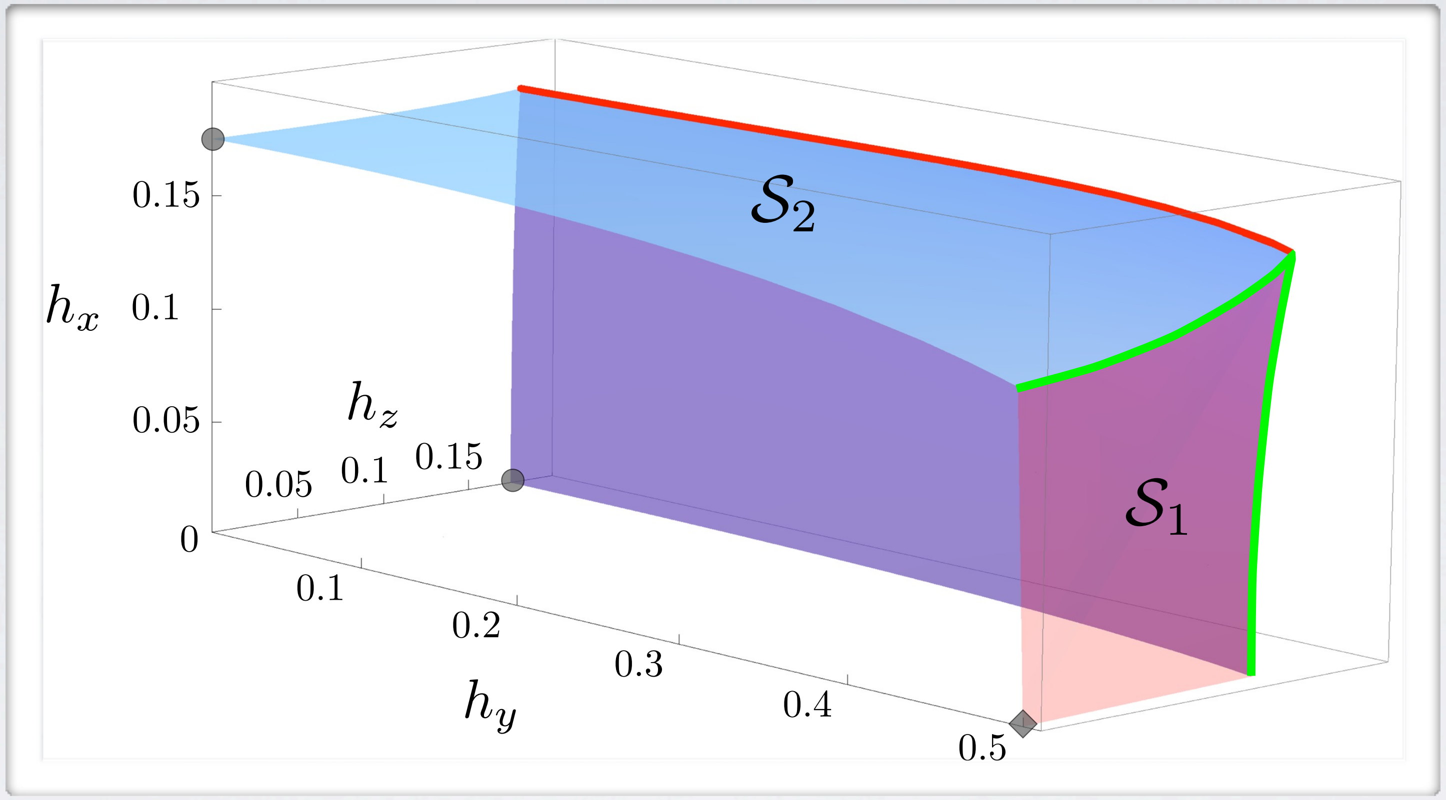
D2 is a logical swap between logical qubits 1 and 2.

Restricted to symmetric subspace of qubits 1 and 2.

FINAL ADIABATIC STATE?

THE PHASE DIAGRAM

Field orientation determines phase transition.

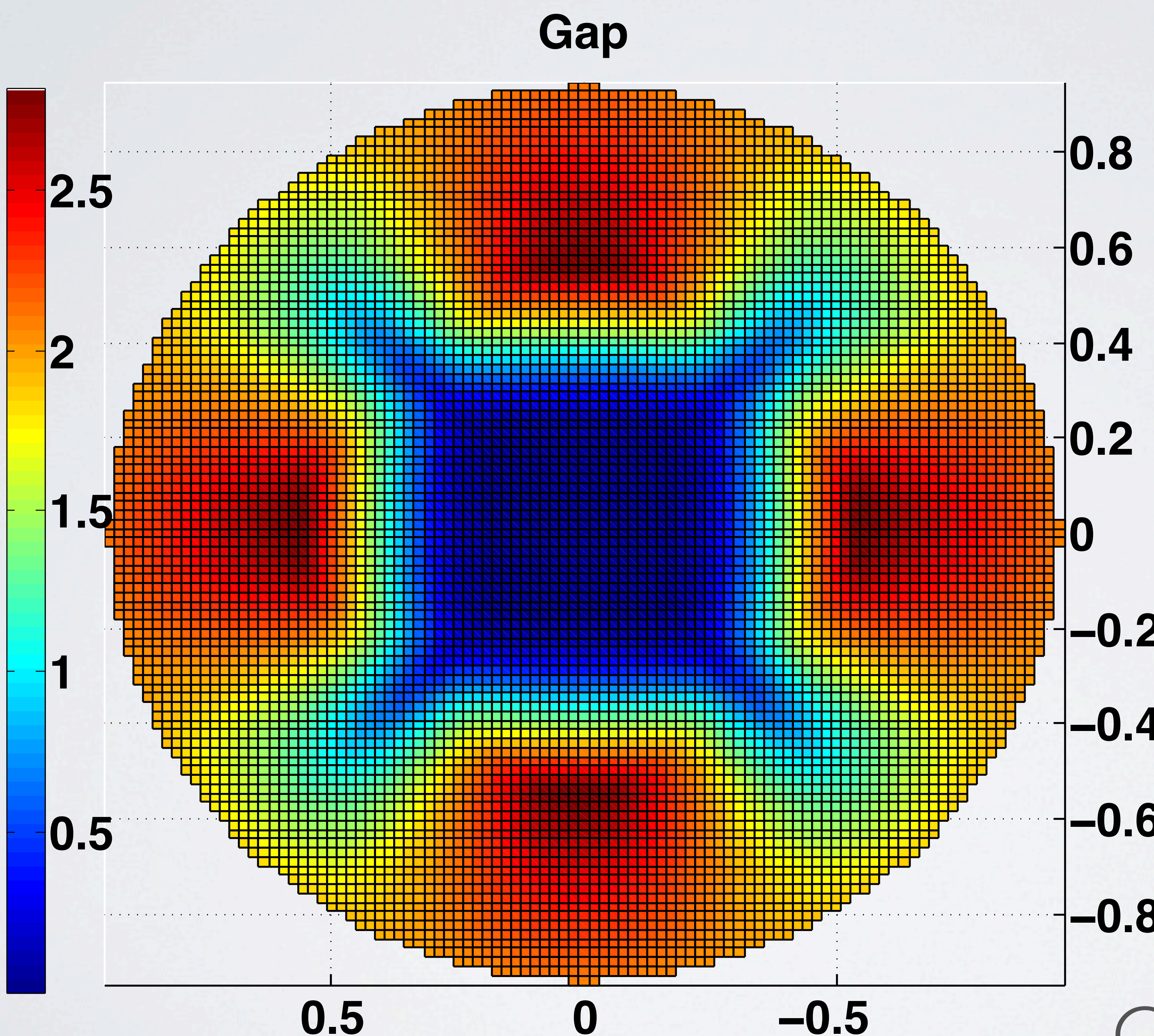


Different anyon species condense.

iPEPS + PCUT numerical study.

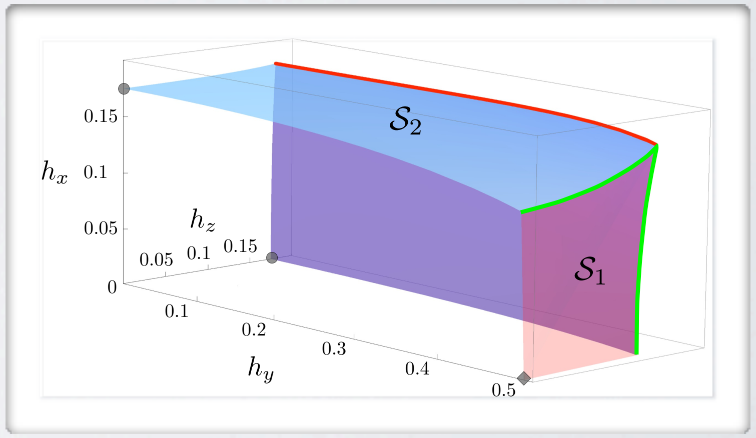
Dusuel, S., Kamfor, M., Orús, R., Schmidt, K. P., & Vidal, J. (2011). Robustness of a Perturbed Topological Phase. Physical Review Letters, 106(10), 107203. doi:10.1103/PhysRevLett.106.107203

NUMERICAL PHASE DIAGRAM



X Field

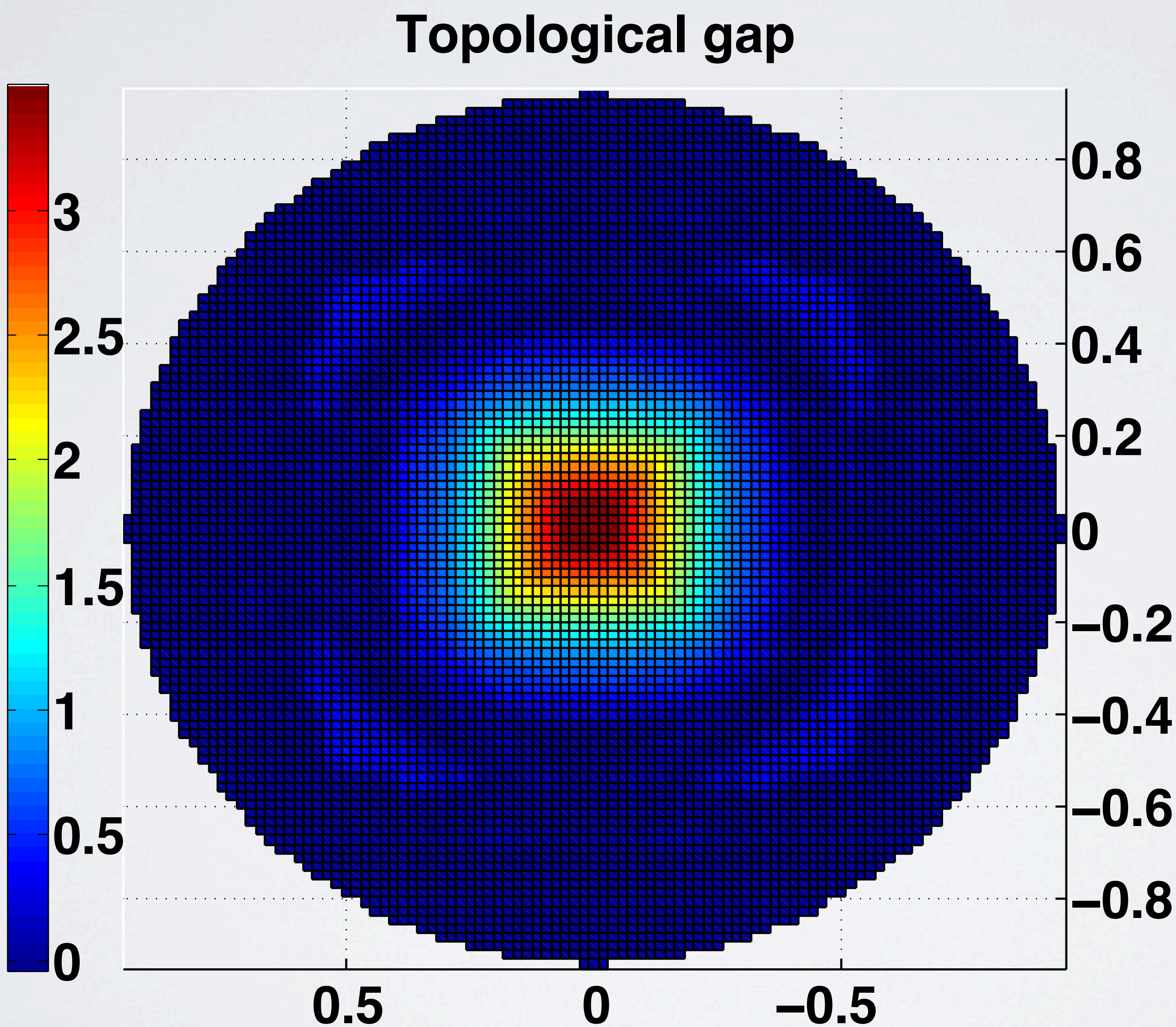
Z Field



Qualitative for L=3

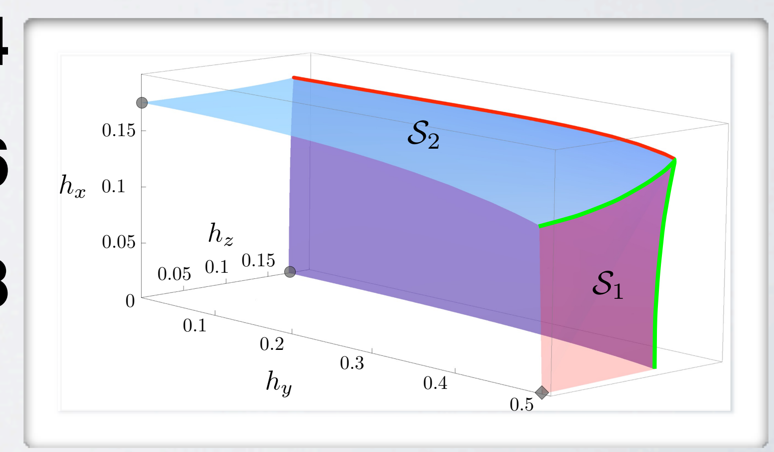
NUMERICAL PHASE DIAGRAM

$E_5 - E_4$



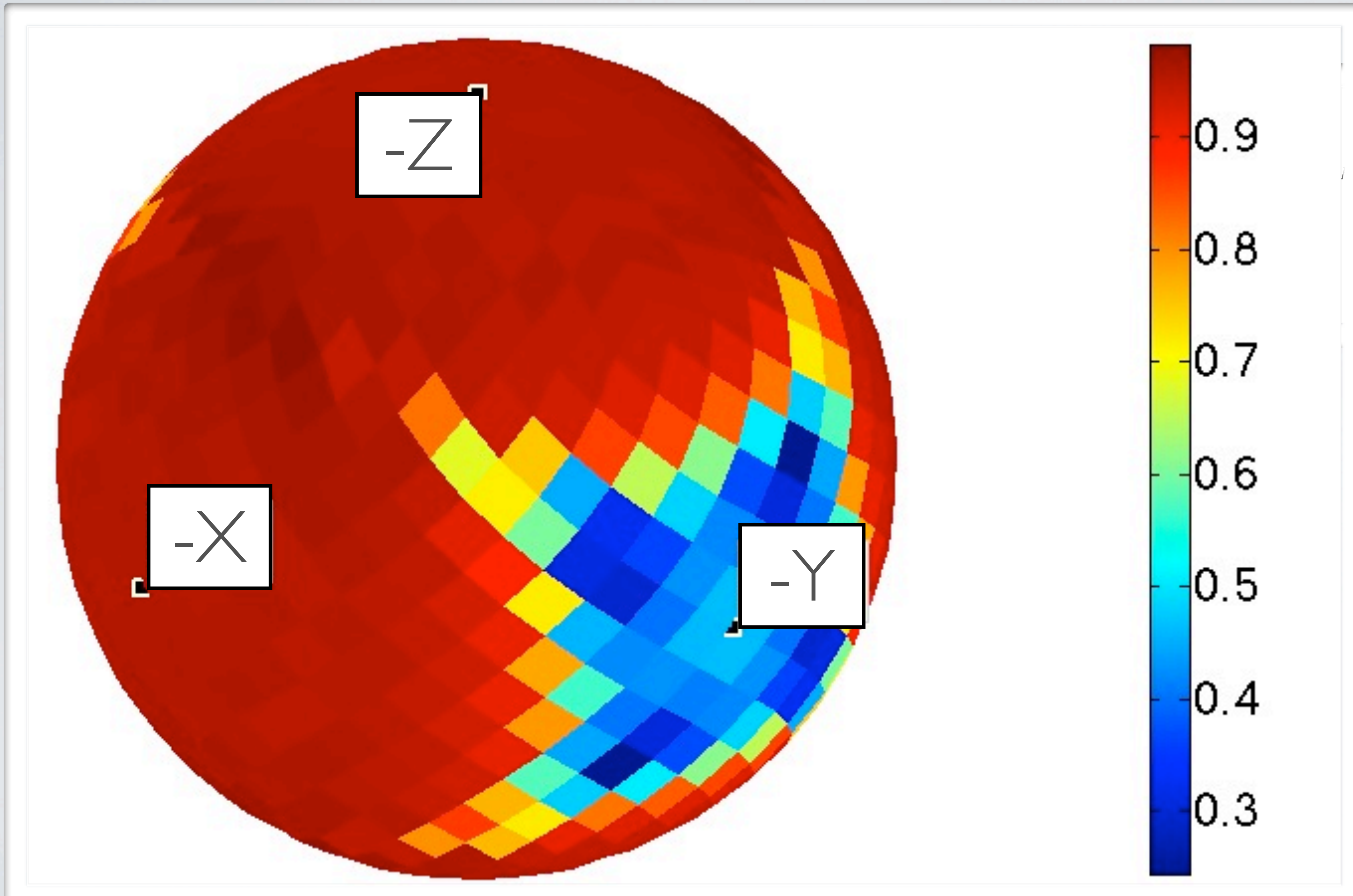
Z Field

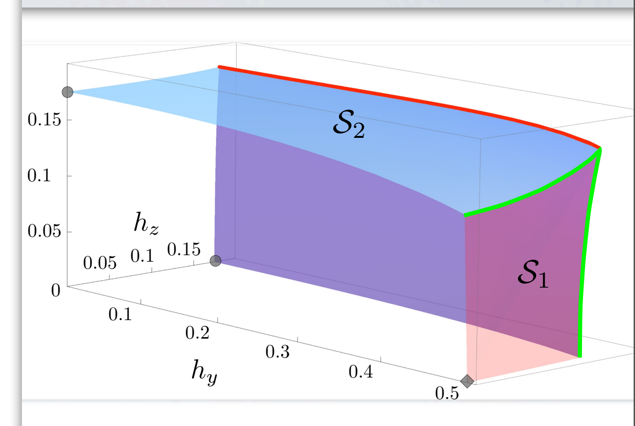
X Field



Qualitative for L=3

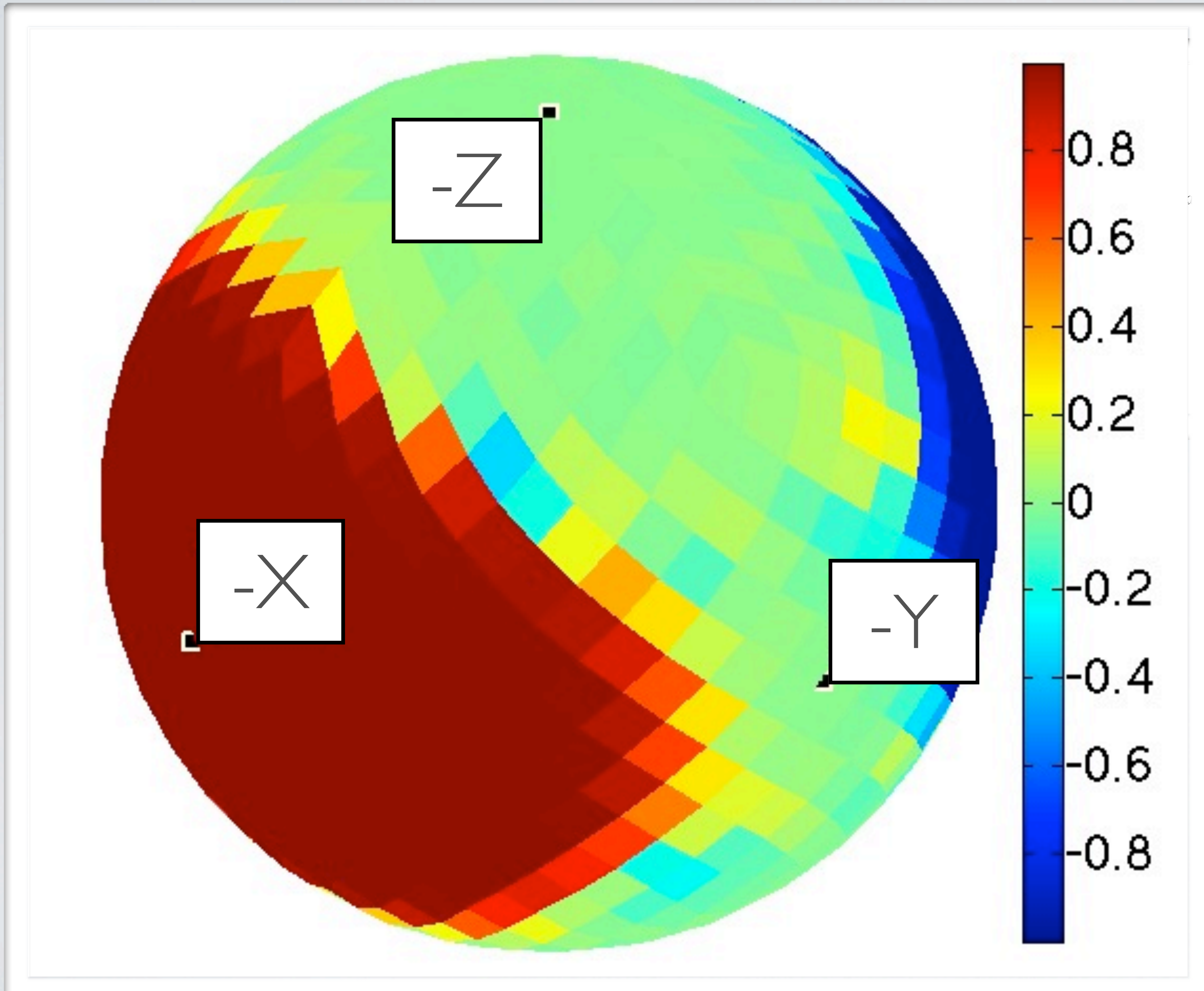
PROBABILITY OF REACHING GROUND STATE SPACE

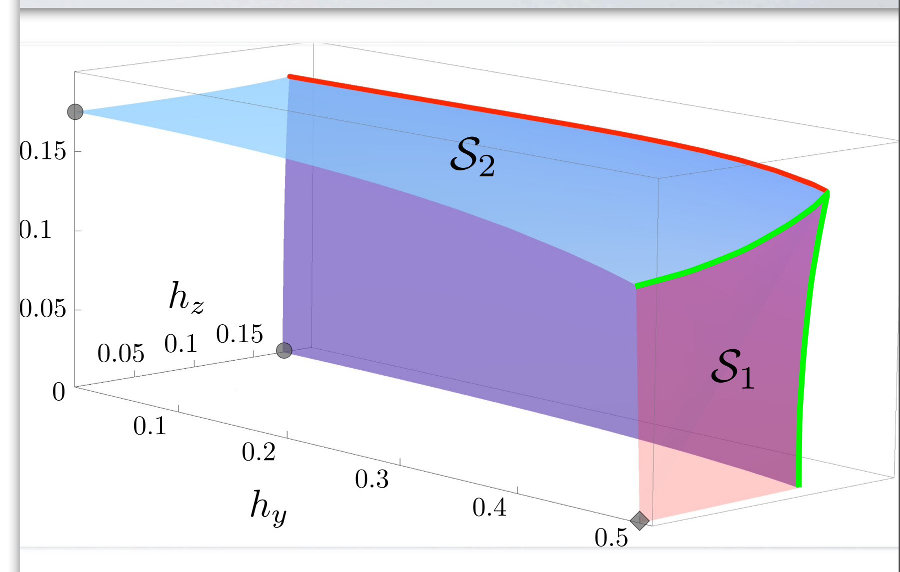




L=3,T=30

LOGICAL X POLARIZATION





L=3,T=30

TWO REGIMES

- 1) Usual adiabatic theorem applicable for the magnon phase.
- 2) Quasi-adiabatic continuation within the topological phase.
  - Unique: parameter space has no obstructions.
  - Exponentially small splitting $\times$ polynomial time = exponentially small non-geometric phase
- Interesting stuff happens at the phase transition.

INFINITE TIME LIMIT

- Infinite time limit adiabatic theorem.
- Ground state gap closes at the end.
- Splitting determined by lowest order perturbation theory.
- For stabilizer codes, order is set by code distance.

$$(H_F + \lambda H_I) \mid n\rangle = E_n \mid n\rangle$$

$$E_n = \sum_{j=0}^{\infty} \lambda^j E_n^{(j)} \qquad\qquad \mid n\rangle = \sum_{j=0}^{\infty} \lambda^j \mid n^{(j)}\rangle$$

Rigolin, G., & Ortiz, G. (2010). Adiabatic Perturbation Theory and Geometric Phases for Degenerate Systems. Physical Review Letters, 104(17), 170406.

DEGENERATE PERTURBATION THEORY

- Perturbative expansion around exact topological point.

$$(H_F + \lambda H_I) \mid n\rangle = E_n \mid n\rangle$$

$$E_n = \sum_{j=0}^{\infty} \lambda^j E_n^{(j)} \qquad \qquad \mid n\rangle = \sum_{j=0}^{\infty} \lambda^j \mid n^{(j)}\rangle$$

- Degeneracy preserved up to order L

$$\lambda^L H_L \approx \left(\frac{\lambda_X}{\Delta}\right)^L \Delta \bar{X} + \left(\frac{\lambda_Z}{\Delta}\right)^L \Delta \bar{Z}$$

OBSERVATIONS

- Preparation of **encoded stabilizer states** is stable
  - To increase in preparation time.
  - To Hamiltonian perturbations.
- Fast prep. time for 2nd order phase transitions (T=poly(L)).
- These are associated to lowest weight logical operators.
- How about other codes?

CAN WE ALWAYS ADIABATICALY PREPARE CODE STATES?

SUPERCOHERENT QUBIT

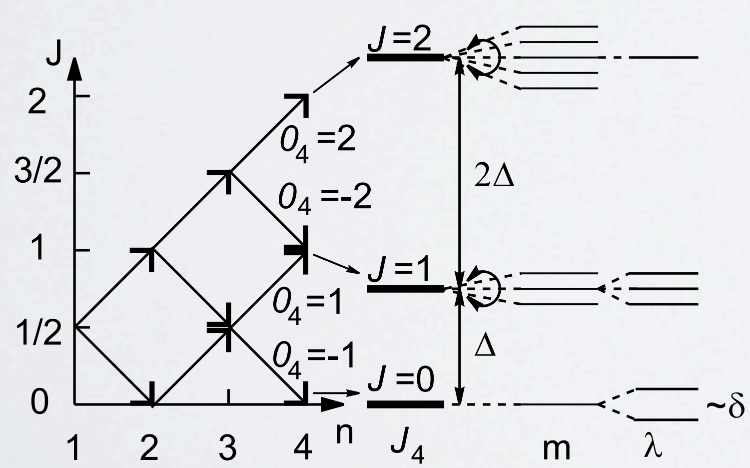
- Decoherence-free subspace for uniform fields and d=2 code.

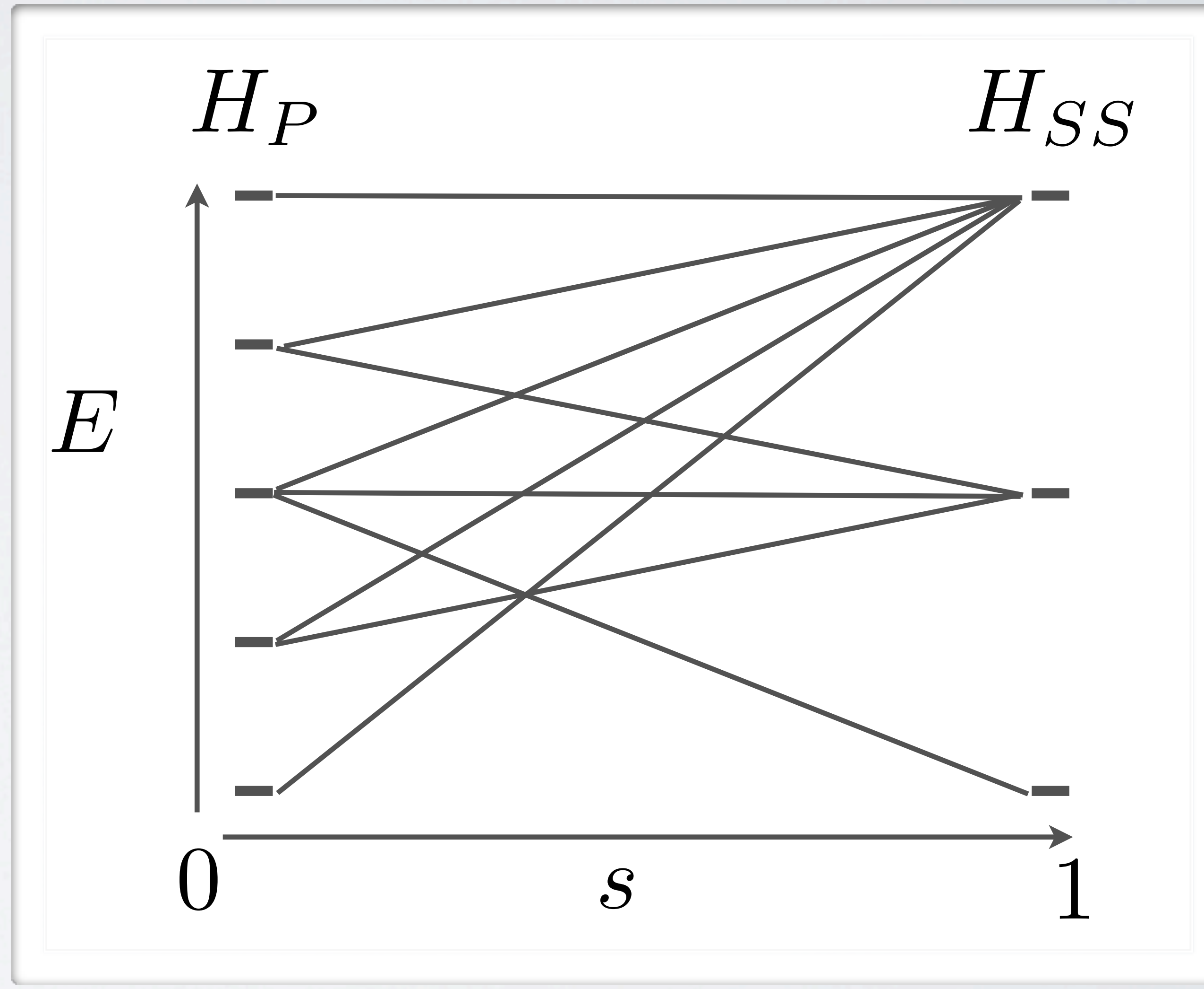
$$H_{SS} = -\sum_{j \neq k = 1}^{4} \vec{\sigma}_j \cdot \vec{\sigma}_k$$

- No avoided crossing

$$[H_{SS}, H_P] = 0$$

- Failure of naïve approach.

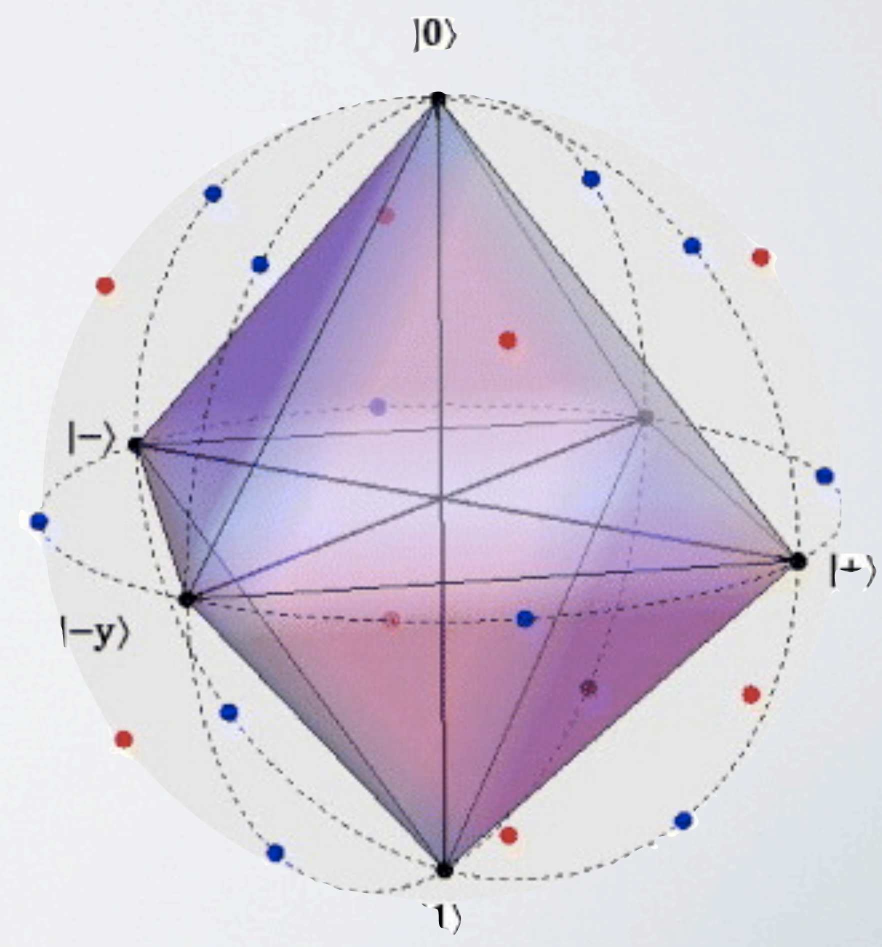




Bacon, D., Brown, K. R., & Whaley, K. B. (2001). Coherence-Preserving Quantum Bits. Physical Review Letters, 87(24), 247902.

CAN OTHER JOINT SYMMETRIES INDUCE OTHER STABLE STATES?

MAGIC STATES?



STEANE'S 7 QUBIT CODE
(COLOR CODES)

$[[7,1,3]]$

$$P = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

4-body terms (stabilizers)

$$H_{CC} = \sum_{p \in \text{ face}} A_p$$

$$A_p = -\bigotimes_{q \in p} X_j - \bigotimes_{q \in p} Z_j - \bigotimes_{q \in p} Y_j$$

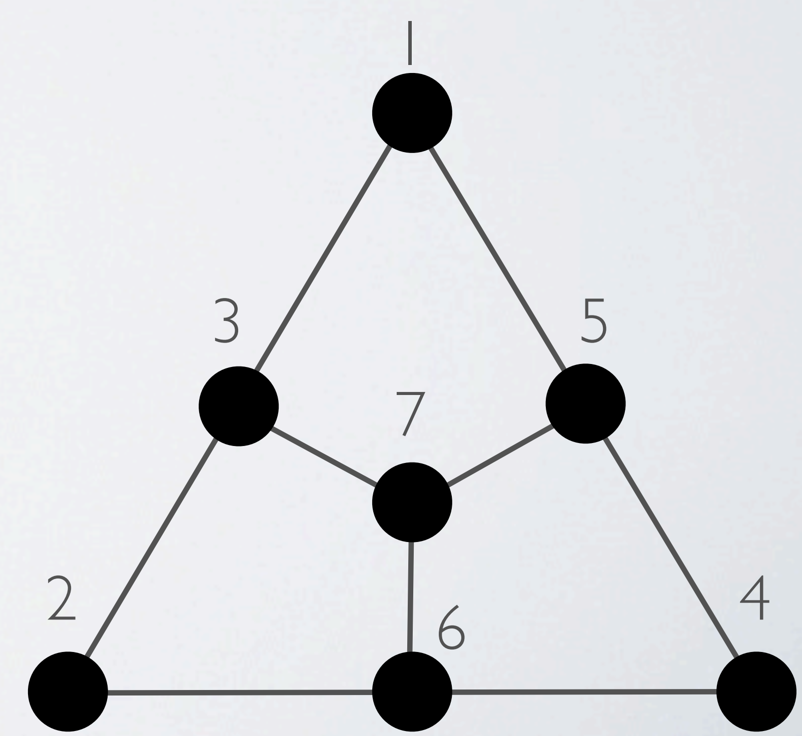
Transverse logical operators.

$$X^{\otimes 7} \equiv -\bar{X} \quad Y^{\otimes 7} \equiv -\bar{Y} \quad Z^{\otimes 7} \equiv -\bar{Z}$$

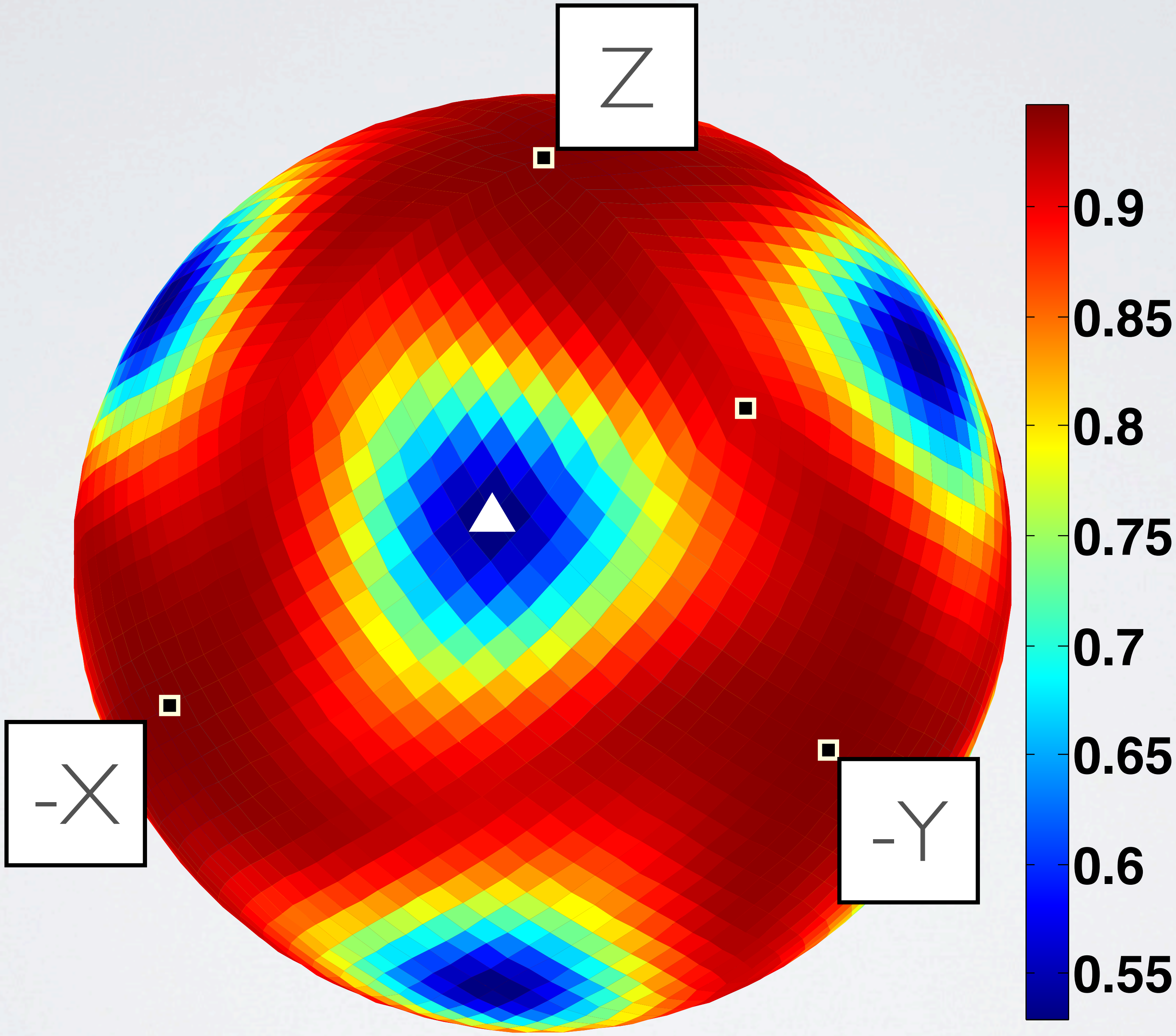
$$H^{\otimes 7} \equiv -\bar{H} \quad R^{\otimes 7} \equiv -\bar{R} \quad R = SH$$

May commute with initial Hamiltonian.

New stable fixpoints?

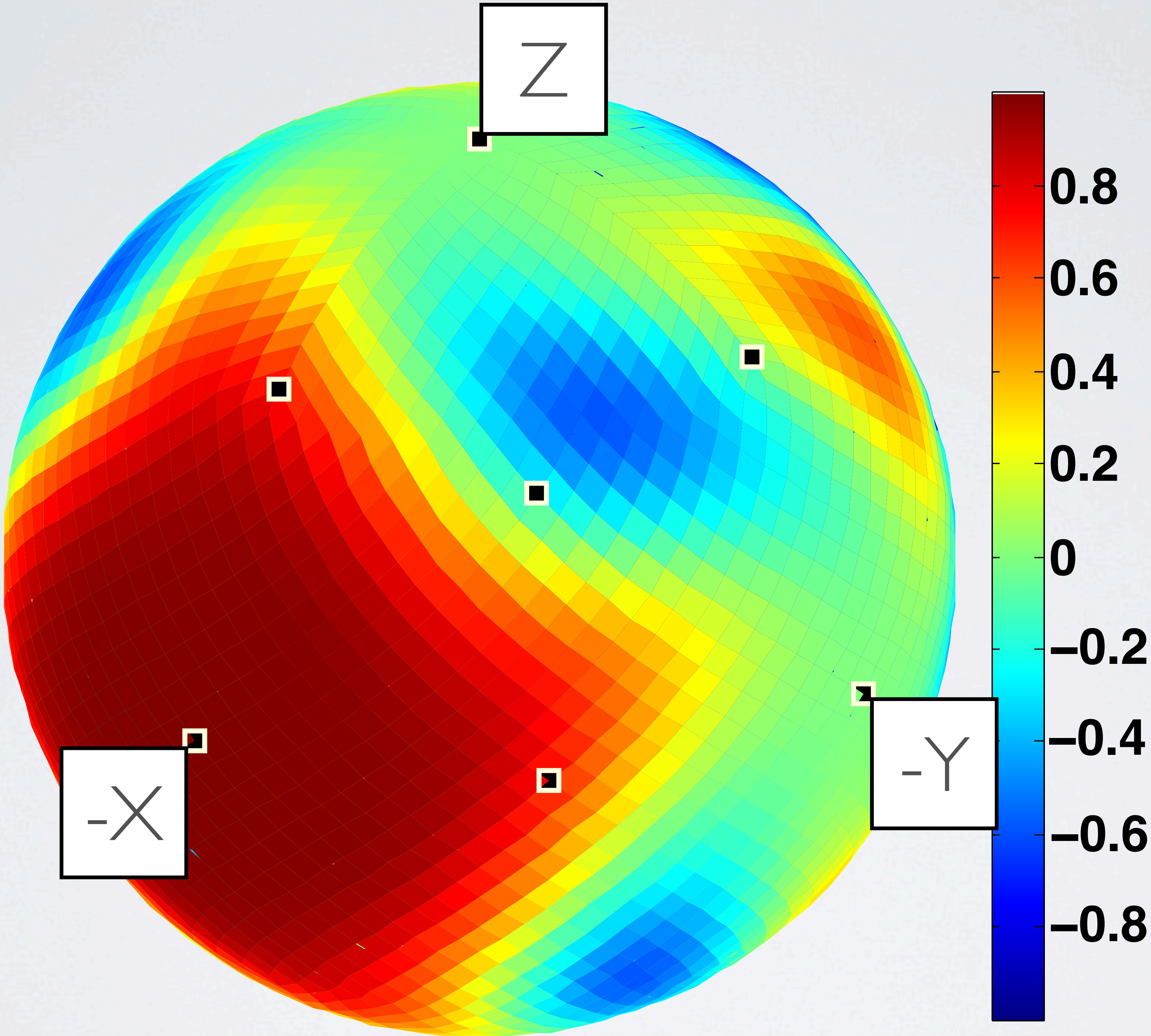


GROUND SPACE OVERLAP



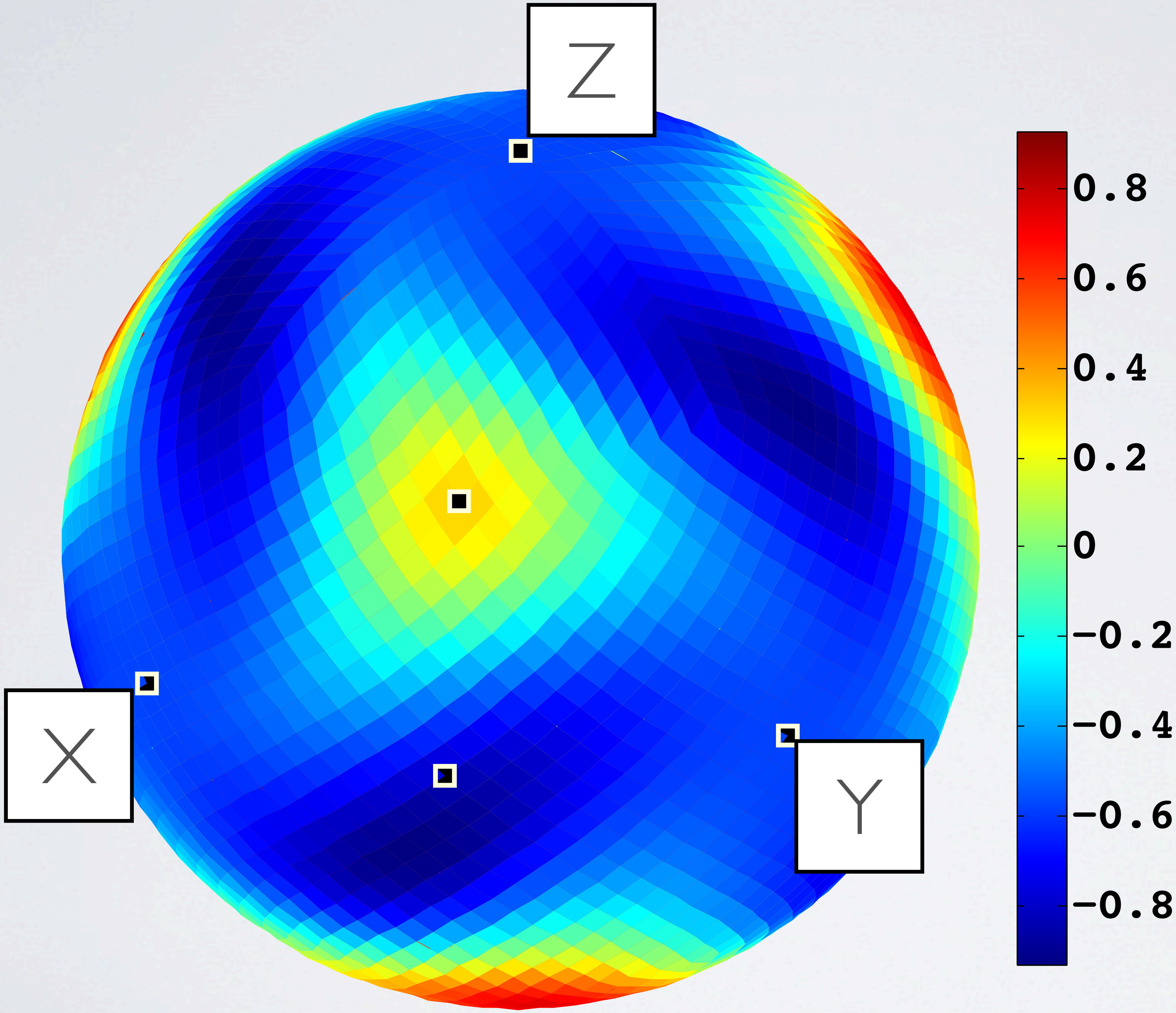
Steane (N=7),T=4

LOGICAL X POLARIZATION



Steane code (N=7),T=4

LOGICAL XYZ POLARIZATION

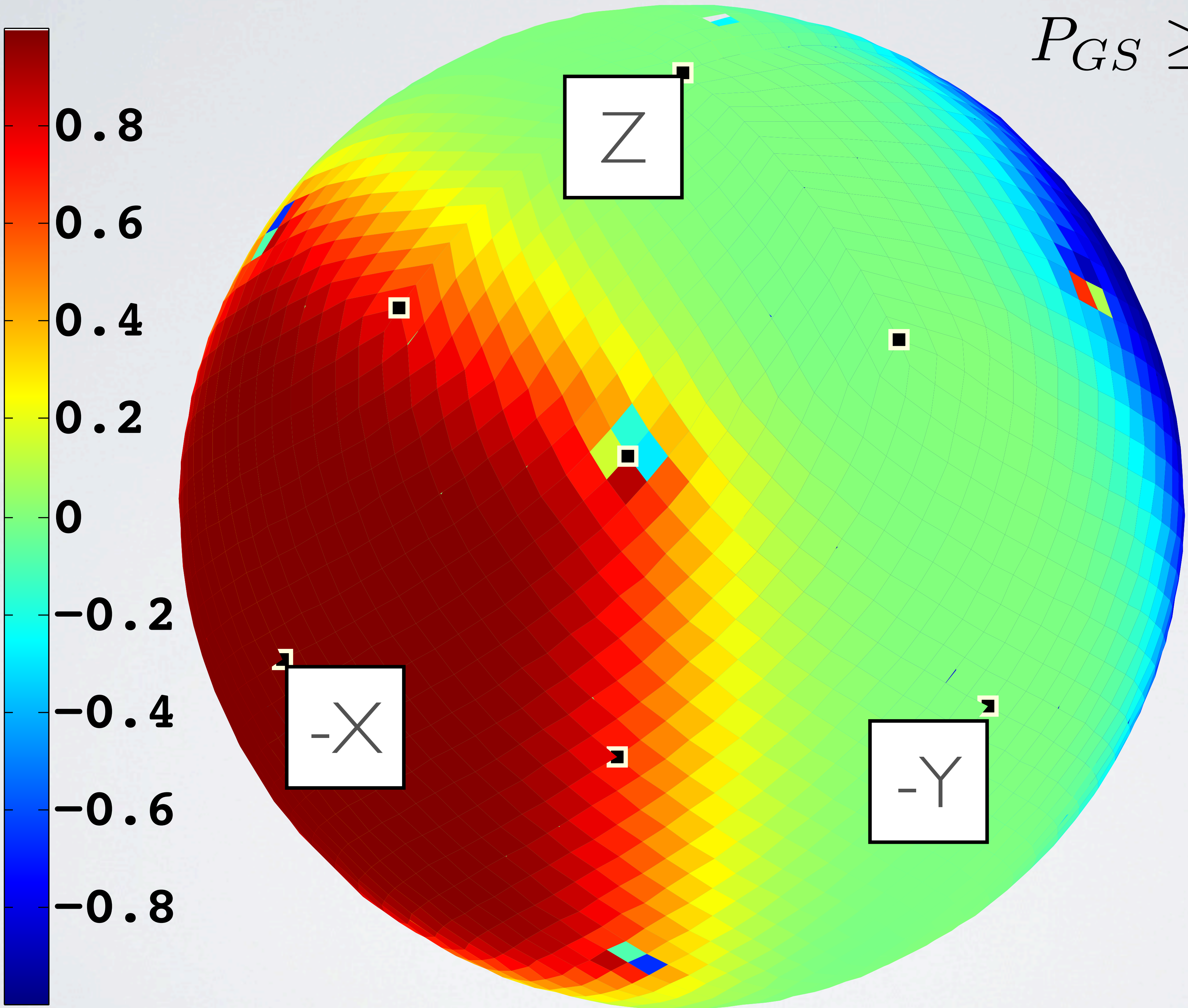


$$\frac{\bar{X}+\bar{Y}+\bar{Z}}{\sqrt{3}}$$

Steane code (N=7),T=4

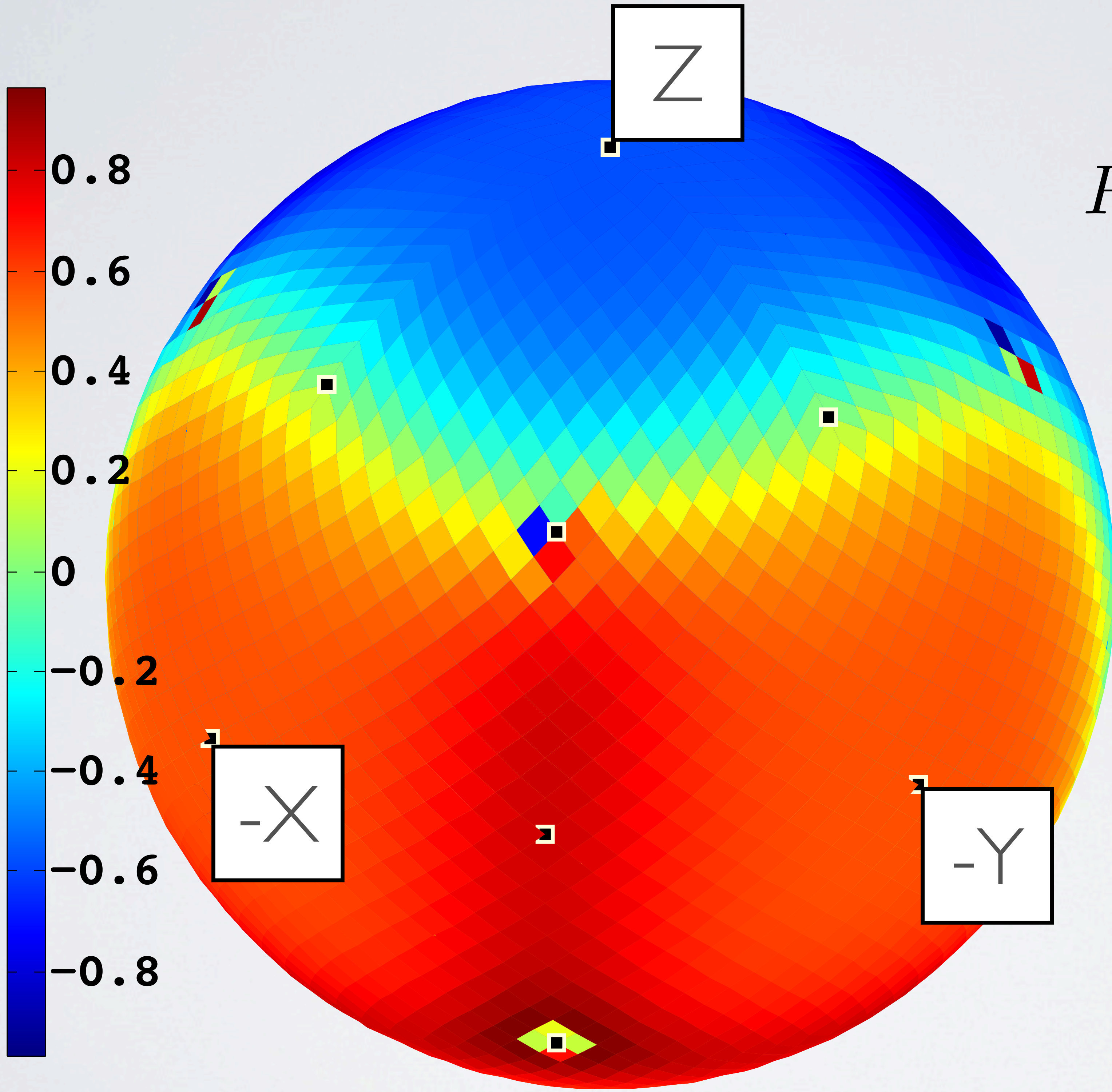
LOGICAL X POLARIZATION

$P_{GS} \geq 1 - 2 \times 10^{-6}$



Steane code (N=7),T=256

LOGICAL XYZ POLARIZATION



$$P_{GS} \geq 1 - 2 \times 10^{-6}$$

$$\frac{\bar{X} + \bar{Y} + \bar{Z}}{\sqrt{3}}$$

Steane code (N=7),T=256

CONCLUSIONS & OUTLOOK

- Efficient preparation of actual code space.
- Resource: encoded ancillas.
  - Robust adiabatic preparation of stabilizer states.
- Joint symmetries are not enough to guarantee stability.
- Can we enhance joint symmetries to prepare "magic states" in a robust adiabatic manner?

THANK YOU!