PROBING THE LATE-TIME EVOLUTION SPECTRUM OF A QUANTUM ANNEALER

Walter Vinci, Paul A. Warburton

12 June

AQC14

OUTLINE

- Quantum annealing as a 'black-box':
  - Use all the information we can possibly extract from a quantum annealing device

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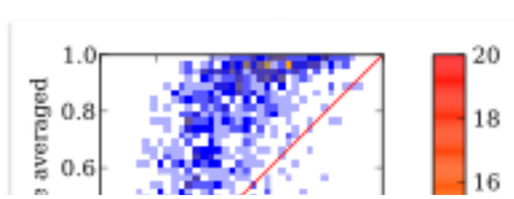
OUTLINE

- Quantum annealing as a 'black-box':
  - Use all the information we can possibly extract from a quantum annealing device
- Non-trivial signature of adiabatic quantum annealing
  - Computable using perturbation theory
- Can we use it to determine the role of entanglement in noisy quantum annealing?
  - Approximate product states
  - Classical spin dynamics (SSSV model)

"SUCCESS PROBABILITY" SIGNATURE (SPS)

[Boixo, Rønnow *et al.* '13 and '14]



- Benchmarking, scaling

noisy quantum annealing?

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- Classical spin dynamics (SSSV model)

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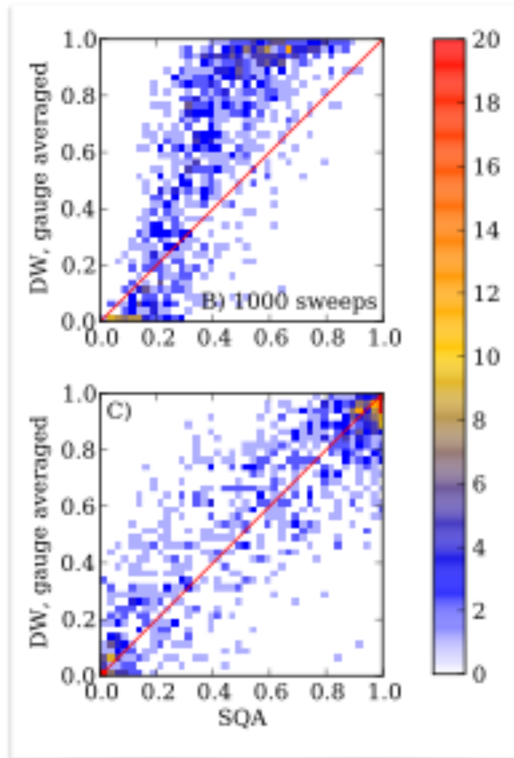


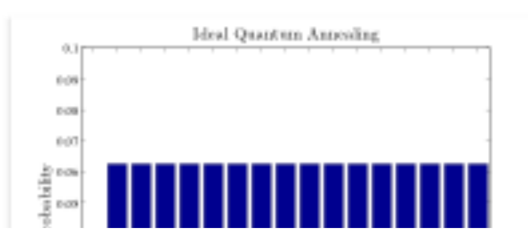
- Benchmarking, scaling
  - Large size limit
  - Unfeasible numerical simulations

- Large scale analysis
  - The D-Wave devices correlate with quantum but not with classical annealing

"GROUND STATE" SIGNATURE (GSS)

- Relative populations of degenerate ground states

[Boixo *et al.* '13; Vinci, Abash *et al.* '14]



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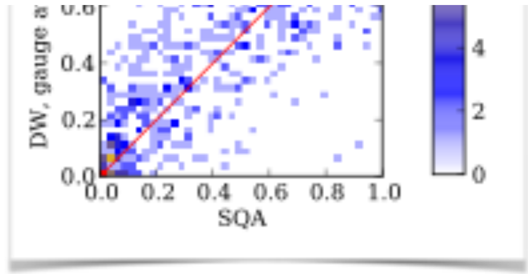
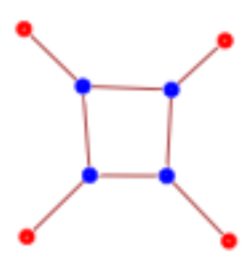


"GROUND STATE" SIGNATURE (GSS)

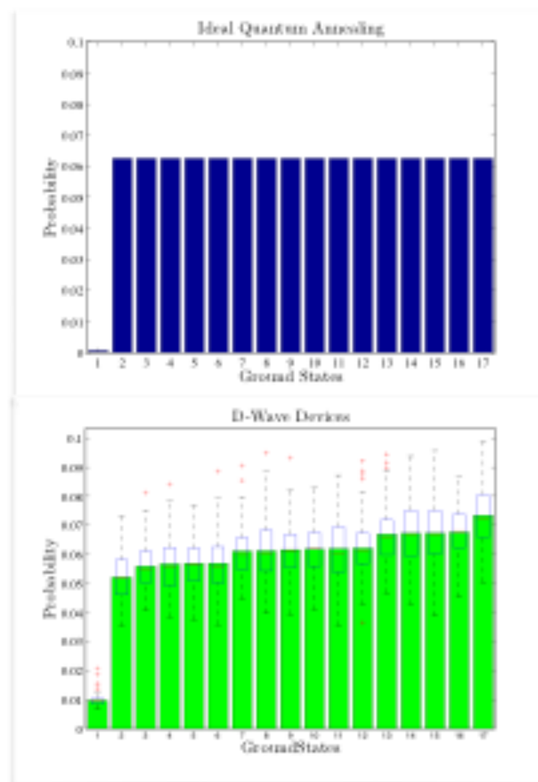
- Relative populations of degenerate ground states

[Boixo *et al.* '13; Vinci, Abash *et al.* '14]

- Single instance analysis:
  - Quantum annealing: first state less probable (suppressed)



$$\Psi_1 = |\uparrow,\uparrow,\uparrow,\uparrow,\uparrow,\uparrow,\uparrow,\uparrow\rangle$$
$$\Psi_G(t_f) = |\downarrow,\downarrow,\downarrow,\downarrow,-,-,-,-\rangle$$
$$|-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$$



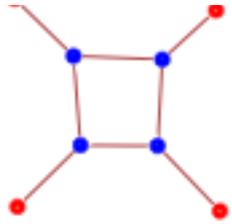
CLASSICAL SPIN DYNAMICS

- Coherent single qubits interacting classically

$$|qubit\rangle \equiv \cos(\theta)|\uparrow\rangle + \sin(\theta)|\downarrow\rangle \rightarrow \vec{M} = (\sin(2\theta), 0, \cos(2\theta))$$

  - Spin-Langevin equation with Landau-Lifshitz friction term

$$\frac{d}{dt}\vec{M}_i = -\left(\vec{H}_i + \vec{\xi}(t) + \gamma \vec{H}_i \times \vec{M}_i\right) \times \vec{M}_i$$



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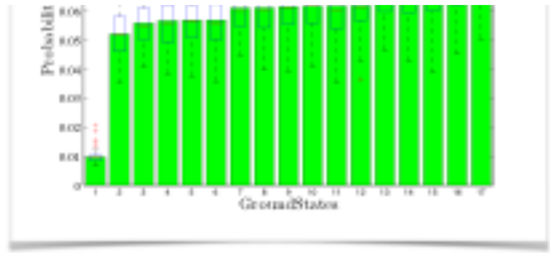

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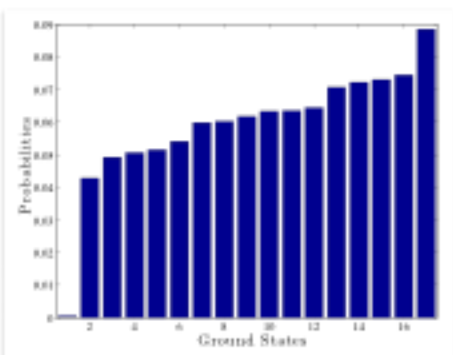
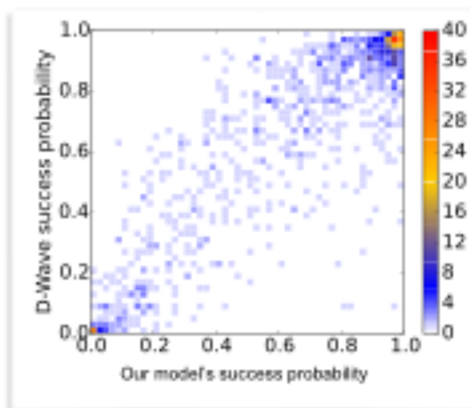
  - Best correlates in the large noise/viscosity regime (Monte Carlo updates)



GSS

SPS



The two signatures are reproduced by the SSSV model

[Shin, Smith, Smolin, Vazirani '13,'14]

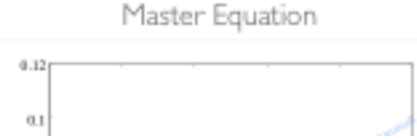
REJECTING SSSV

- SSSV can be rejected as an accurate description of the D-Wave devices
  - Open-system quantum dynamics works better

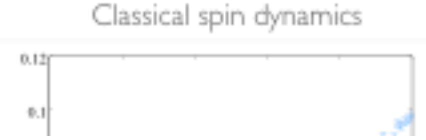
[Albash, Vinci et *al.* '14]

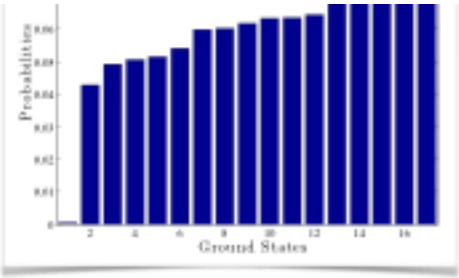
D-Wave

Master Equation



Classical spin dynamics





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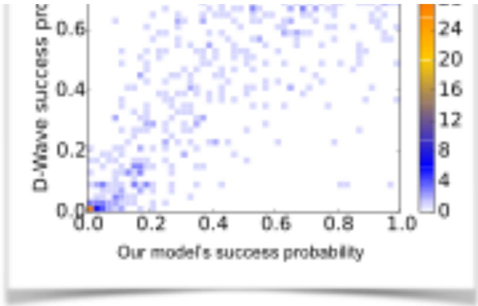

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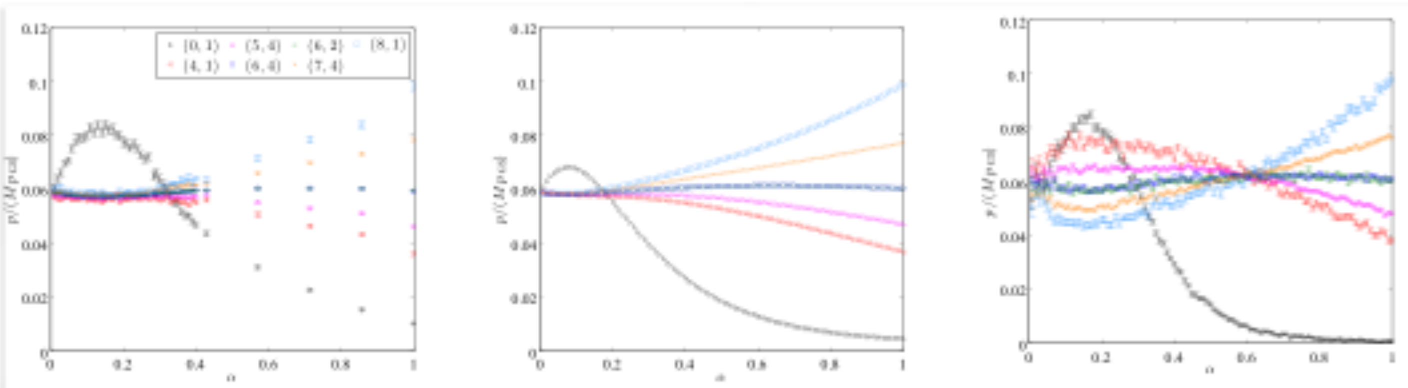
D-Wave Master Equation Classical spin dynamics



- SSSV has to be fine-tuned to reproduce the GSS:
  - Does it fail on other instances?
  - If so, does entanglement play a crucial role?

STRATEGY

Study ground state signature on a very large number of small instances

- Success probability signature

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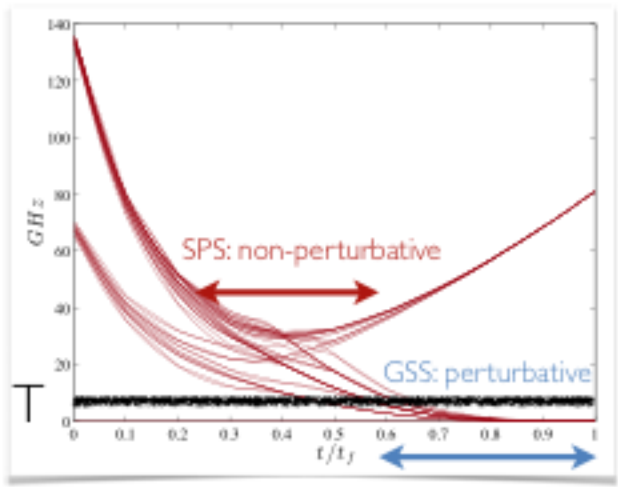
STRATEGY

Study ground state signature on a very large number of small instances



- Success probability signature
  - Non-perturbative origin
  - Trivial for small problems
- Ground state signature
  - Direct connection to the late-time evolution spectrum
  - Non-trivial in the limit of ideal quantum annealing

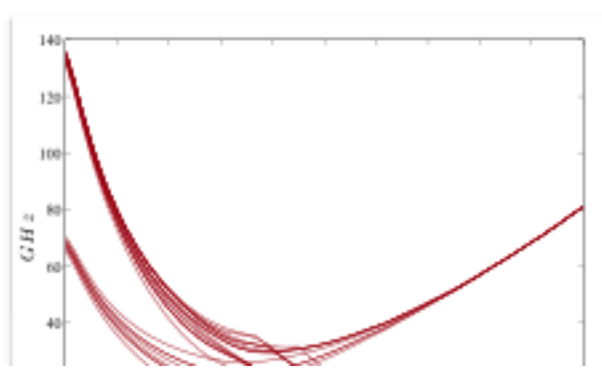
PERTURBATION THEORY

- Ideal quantum annealing

$$P_i^{E_0} = \lim_{t \to t_f} |\langle \Psi_i^{E_0} | \Psi_G(t) \rangle|^2$$

- Annealing schedule

$$H(t) = A(t) H_P + B(t) H_1$$



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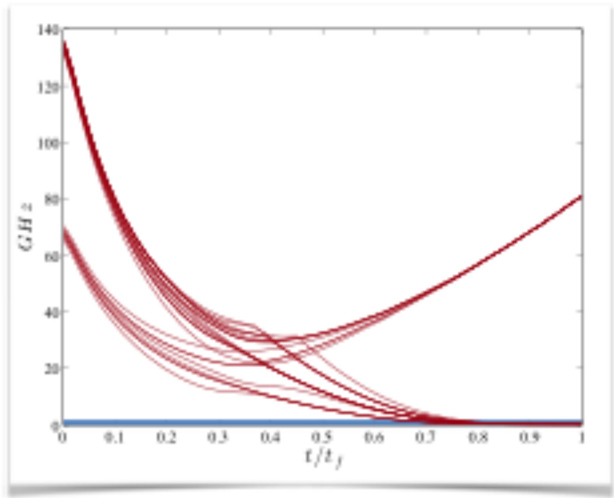

- Annealing schedule

$$H(t) = A(t) H_P + B(t) H_1$$

- Second order degenerate perturbation theory

$$H_I = B(t) \sum_i \sigma_i^x \qquad \delta E_\alpha \simeq \epsilon(t) (H_I)_\alpha + \epsilon(t)^2 (H_I^2)_\alpha$$

$$\Psi_\alpha \simeq \sum_i a_\alpha^i \Psi_i, \quad P_i^{E_0} = |a_0^i|^2$$ 'Zeroth' order

NON-ADIABATICITY

- Exact ground state not protected by a minimum gap
  - GSS signature can be heavily affected
- Include first excited states:
  - Exponentially decaying 'activation

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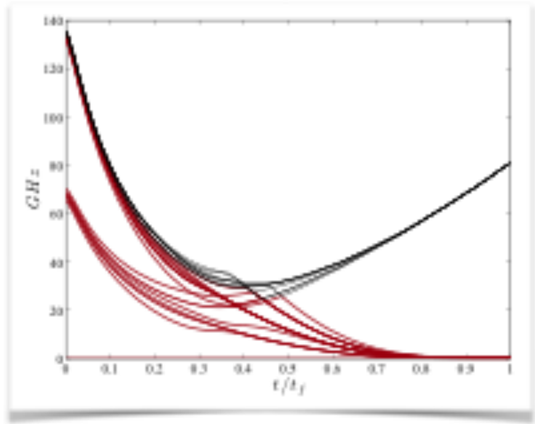
NON-ADIABATICITY

- Exact ground state not protected by a minimum gap
  - GSS signature can be heavily affected

- Include first excited states:
  - Exponentially decaying 'activation rate' of both dynamical and thermal excitations
  - Thermal-like distribution



$$\rho_{\alpha\alpha}^{phen}(\mathcal{T}_{eff}) = \frac{e^{-\epsilon_\alpha/\mathcal{T}_{eff}}}{\sum_\beta e^{-\epsilon_\beta/\mathcal{T}_{eff}}} \qquad P_i^{th}(\mathcal{T}_{eff}) = \Psi_\alpha^i \, \rho_{\alpha\beta}^{phen}(\mathcal{T}_{eff}) \, \Psi_\beta^{i\dagger}$$

EFFECTIVE TEMPERATURE

- A 'best-fit' effective temperature is obtained for each instance which minimizes the following:

$$N(\mathcal{T}_{eff}) = \frac{1}{n_0} \sum_{i=1}^{n_0} \frac{|P_i^{th}(\mathcal{T}_{eff}) - P_i^{exp}|}{P_i^{th}(\mathcal{T}_{eff}) + P_i^{exp}}$$

  - Instance-dependent phenomenological measure of non-adiabatic effects

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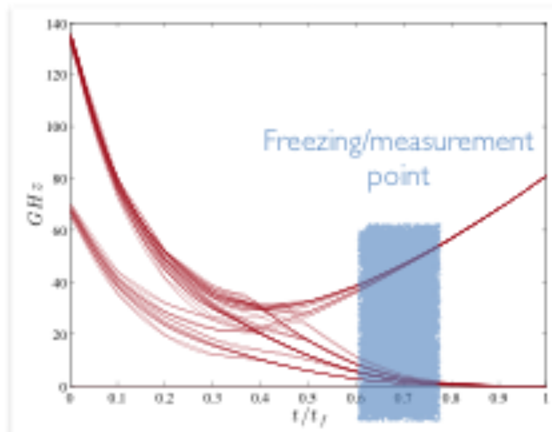
  - Instance-dependent phenomenological measure of non-adiabatic effects

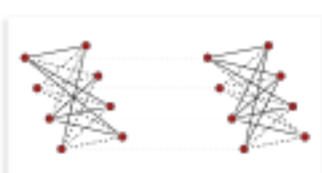
- Freezing point before the end of the adiabatic evolution
  - Optimization analysis does not depend on the position of the freezing point



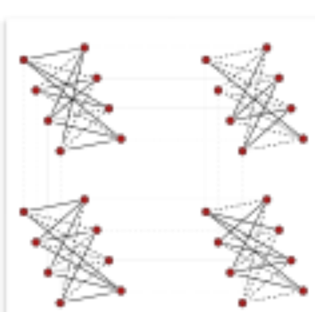
EXPERIMENTAL METHODS

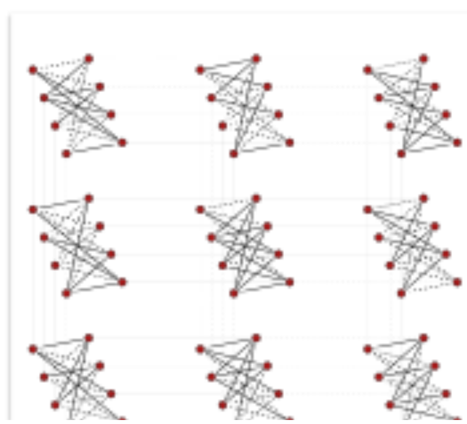
1000 randomly generated instances on NxM Chimera graphs

$$H_{\mathrm{P}} = \sum_{i,j} J_{ij} \sigma_i^z \sigma_j^z \qquad J_{ij} = \pm 1$$

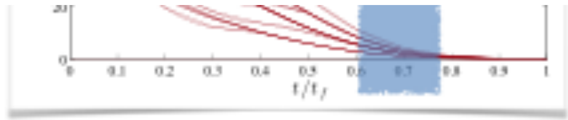


Solid: anti-ferro





depend on the position of the freezing point



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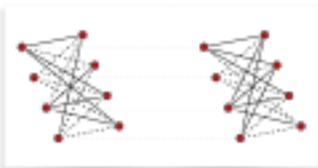
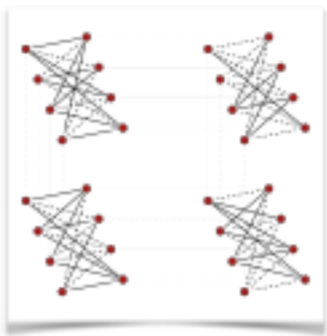
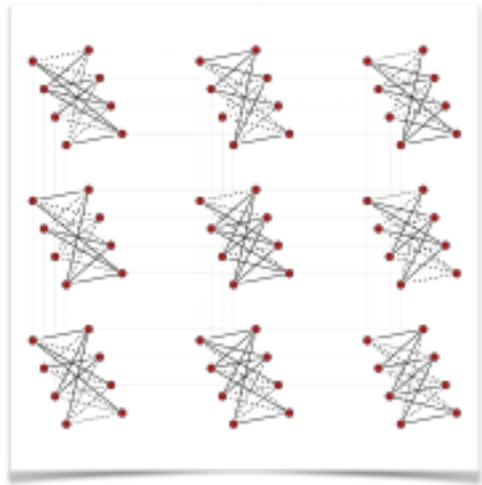






Solid: anti-ferro
Dashed: ferro

- Experimental advantages in having vanishing longitudinal fields
  - Avoid problematic calibration (cross-talk effects) $|J| \simeq |h|$
  - Parity symmetry to estimate experimental errors

$$|\uparrow_i\rangle \longleftrightarrow |\downarrow_i\rangle \quad \delta P_i \simeq |P_i - P_i'|$$

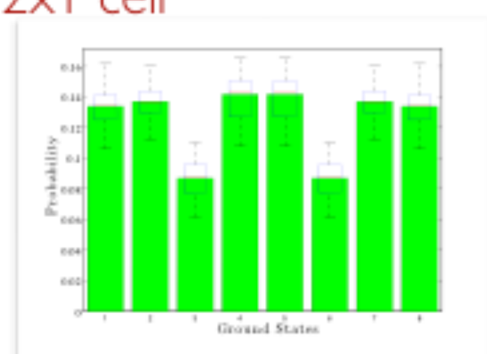
EXAMPLES: GOOD FIT

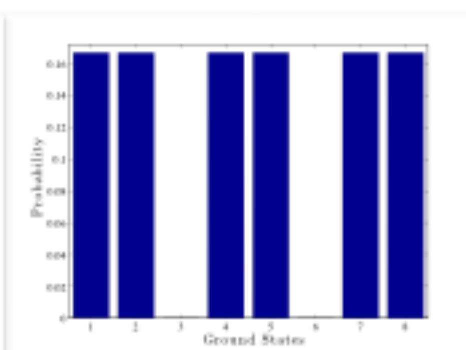
Experimental

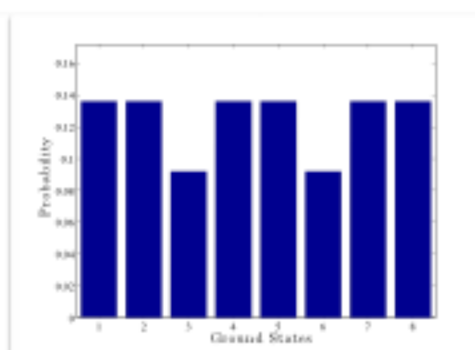
Exact ground state

Thermal best fit

2x1 cell







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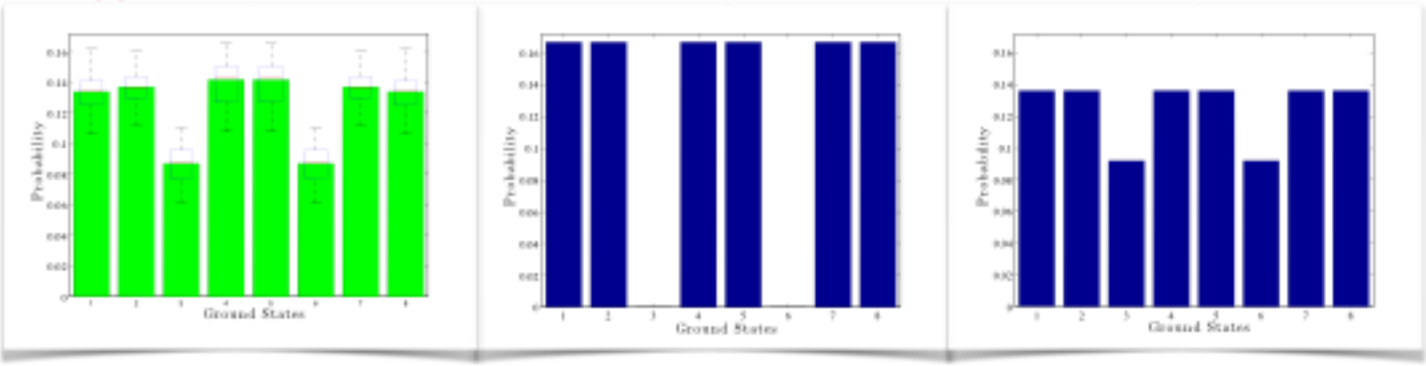
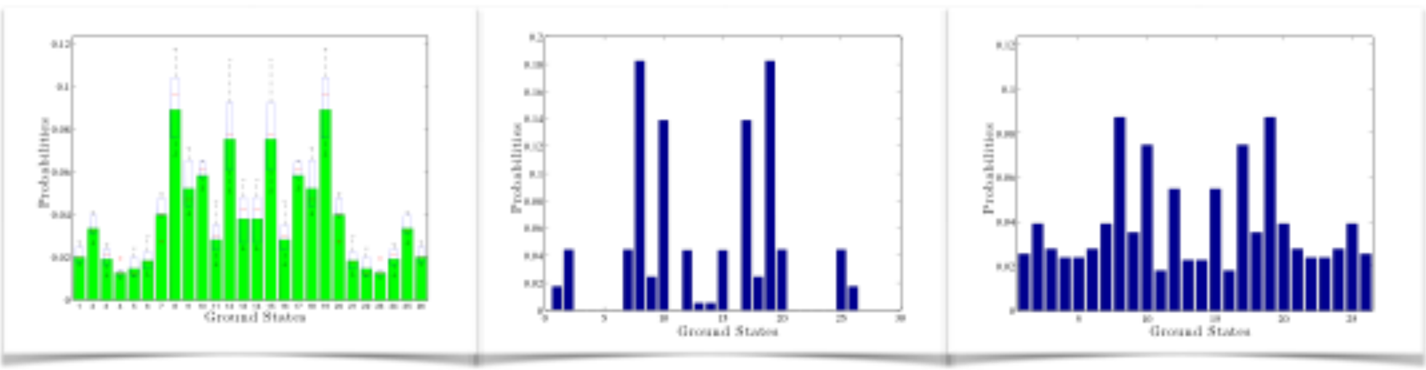


2x2 cell



EXAMPLES: BAD FIT

Experimental | Exact ground state | Thermal best fit

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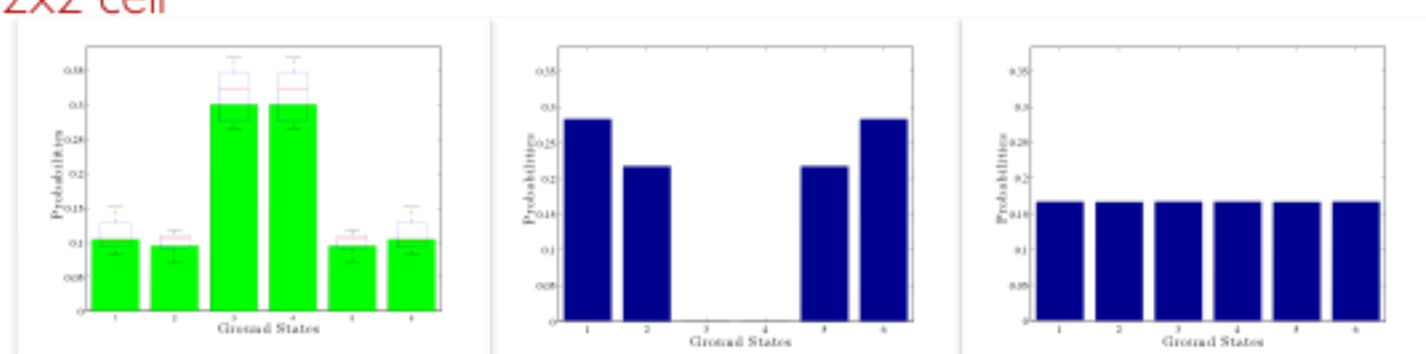

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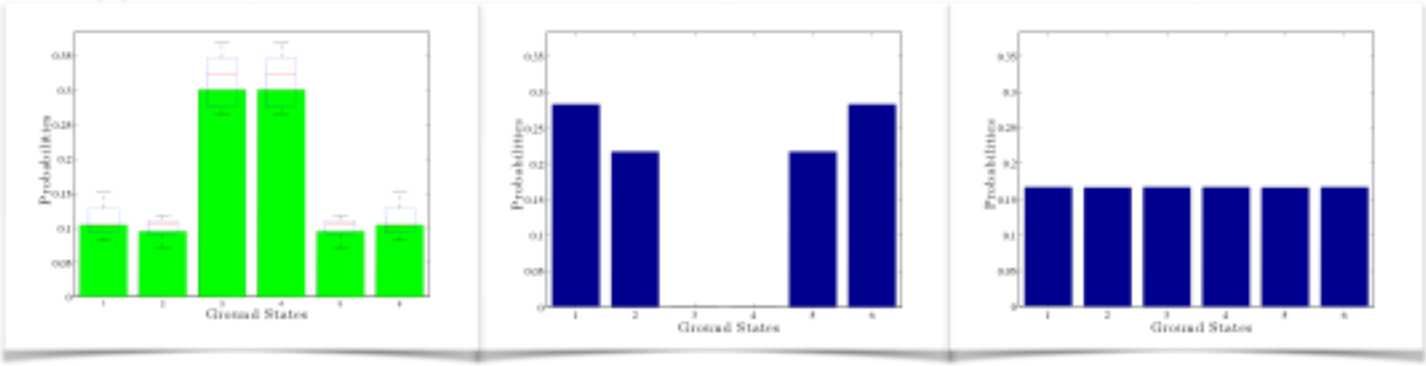
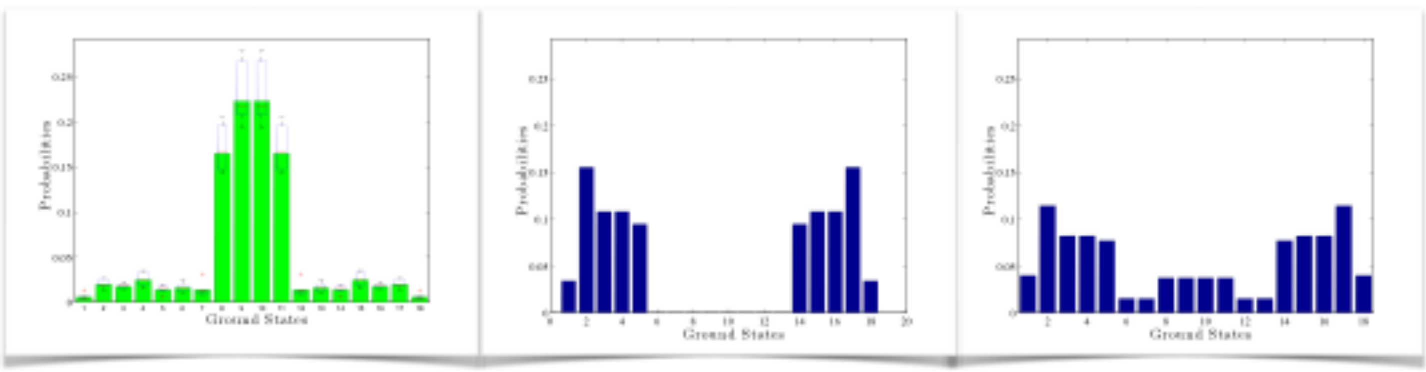
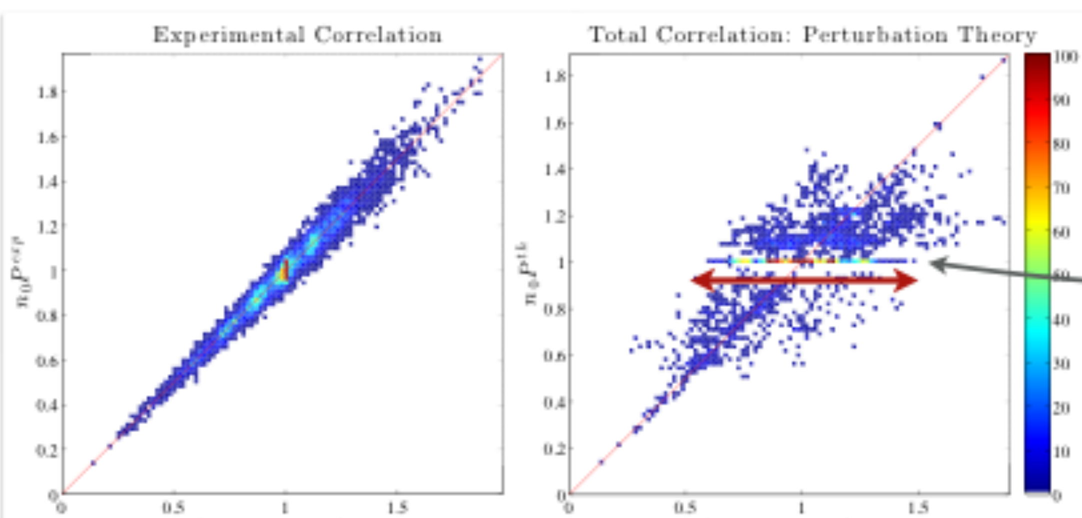


3x3 cell



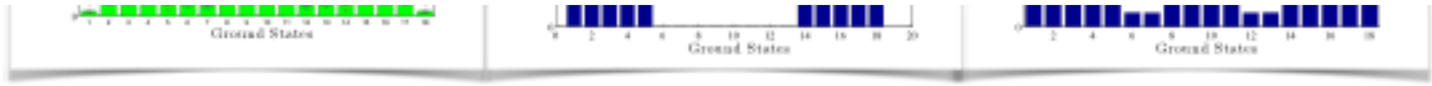
CORRELATIONS: 2X1 CELL



Perturbative analysis:

- 711 trivial
- 3 bad fit

Indirect measurement of non-adiabatic excitations



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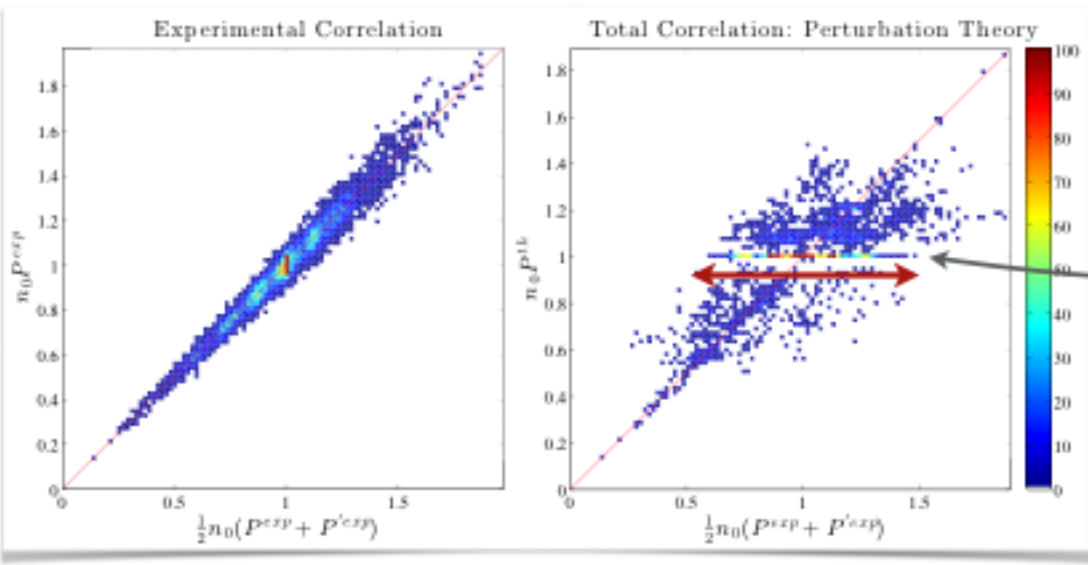

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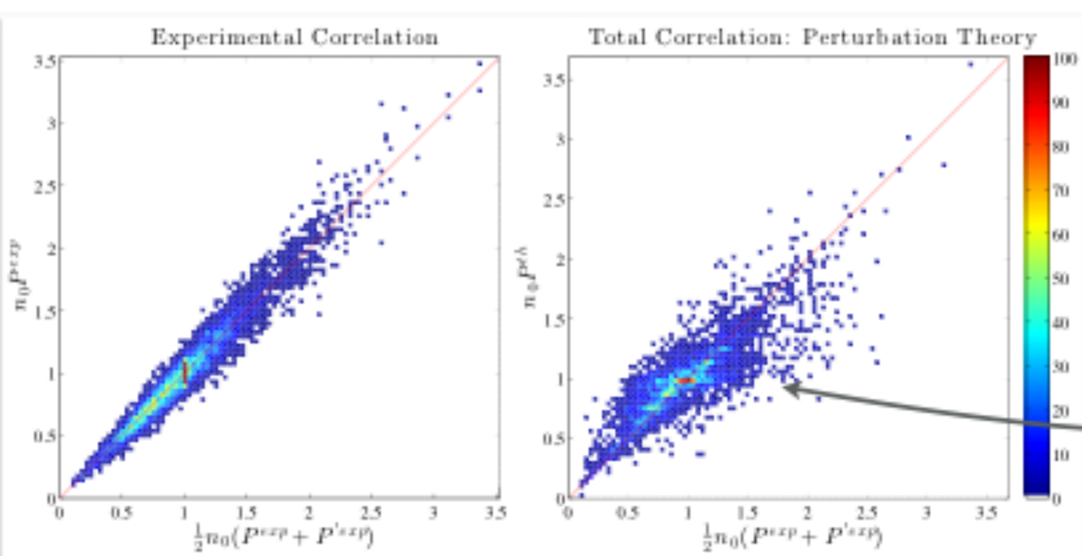
Indirect measurement of non-adiabatic excitations

- Experimental suppression (enhancement): $n_0 P_i^{exp} < 1 \quad (n_0 P_i^{exp} > 1)$
- Theoretical suppression (enhancement): $n_0 P_i^{th} < 1 \quad (n_0 P_i^{th} > 1)$

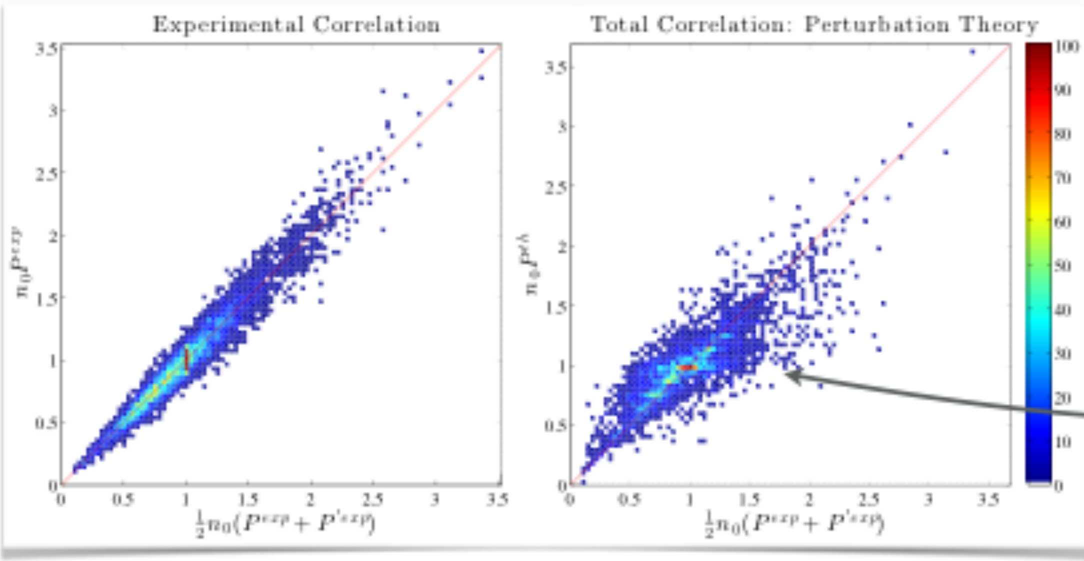
CORRELATIONS: 2X2 CELL



Perturbative analysis:

- 386 trivial
- 8 bad fit

CORRELATIONS: 2X2 CELL



Perturbative analysis:
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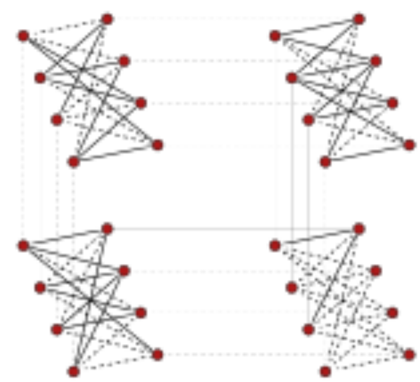
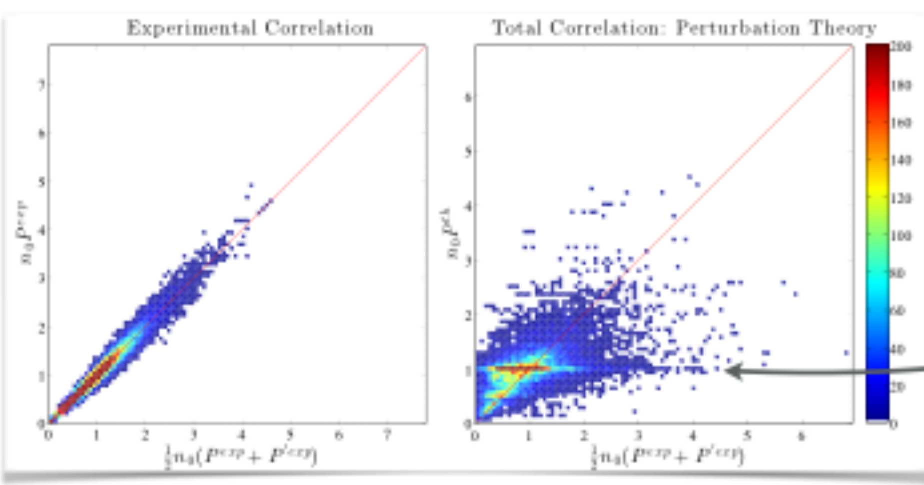
- The connectivity of all spins is 5 (odd):
  - No feee spins $E(|\uparrow\rangle_i) = E(|\downarrow_i\rangle)$
  - No first-order perturbative corrections

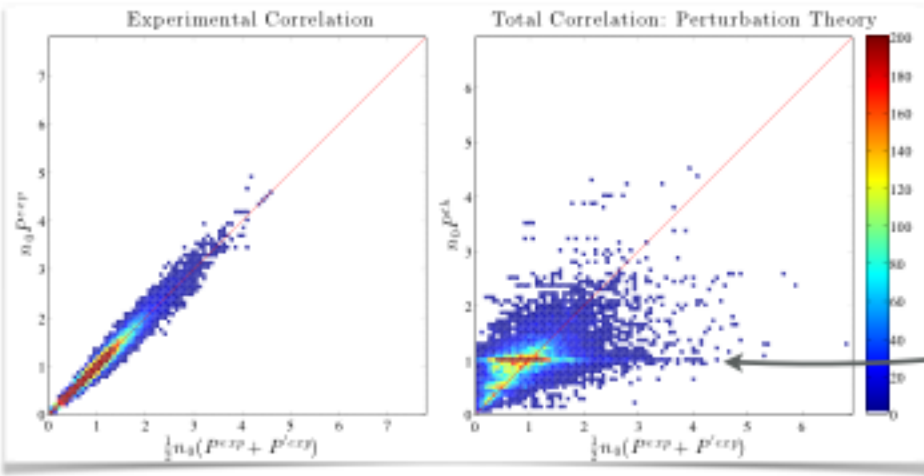


CORRELATIONS: 3X3 CELL



Non-perturbative (gap) excitations dominate the ground state signature
- 255 trivial
- 103 bad fit

CORRELATIONS: 3X3 CELL



Non-perturbative (gap) excitations dominate the ground state signature

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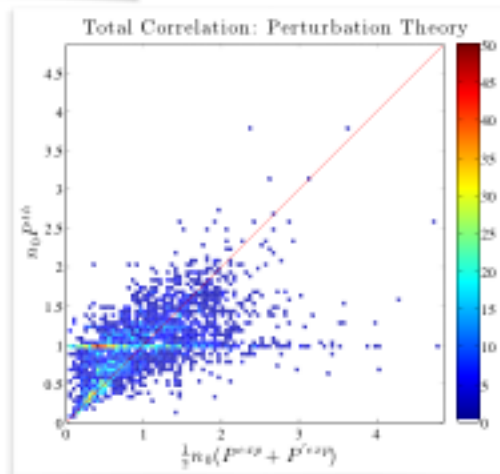
Subset of 300 instances:

- Lowest degeneracy
- Strongest GSS



PRODUCT ANSATZ

- The ground state (exact spectrum) is generically entangled
  - Is the observed correlation a signature of entanglement?

- Product ansatz:

$$\Psi = \prod_i \left(\cos(\theta_i)|\uparrow\rangle_i + \sin(\theta_i)|\downarrow\rangle_i\right)$$

$$E = A(t)\sum_i \sin(2\theta_i) + B(t)\left(\sum_i h_i\cos(2\theta_i) + \sum_{i,j} J_{i,j}\cos(2\theta_i)\cos(2\theta_j)\right)$$

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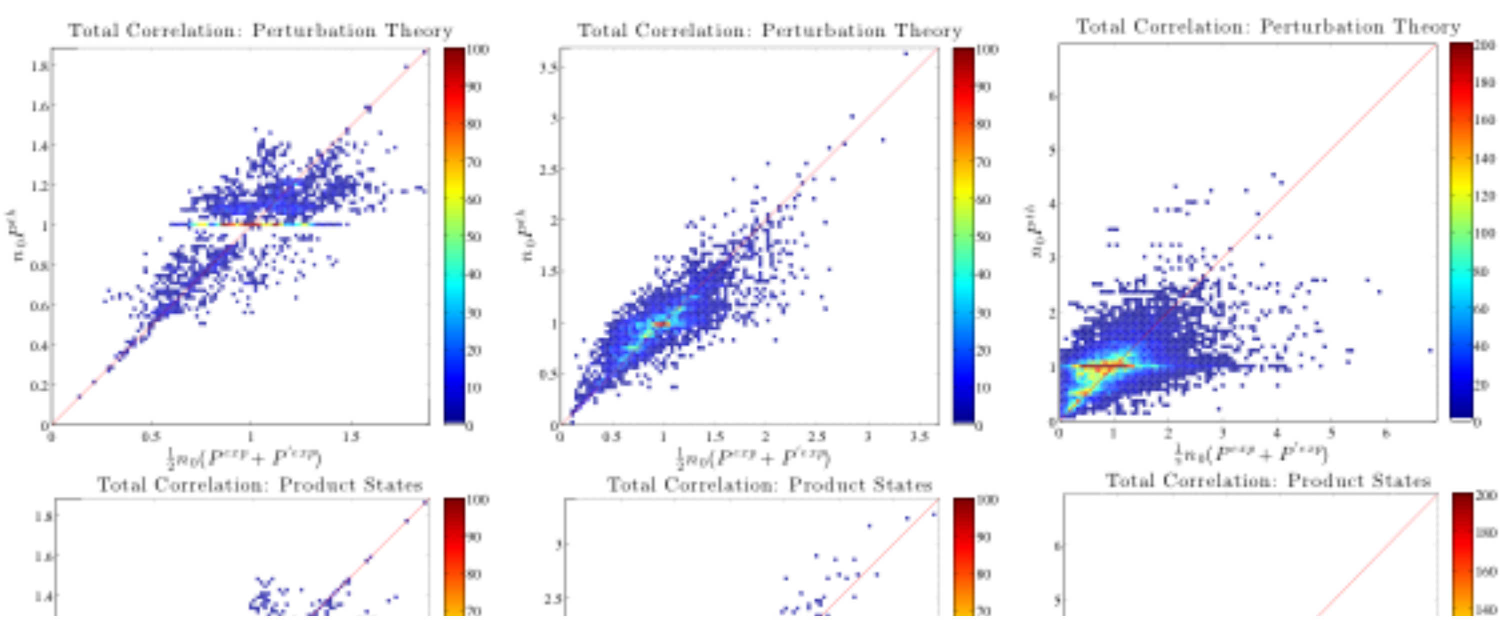
- Approximate ground state:
  - Minimization over the N angles

- Inclusion of 'excited states':
  - Boltzmann sampling over the N angles

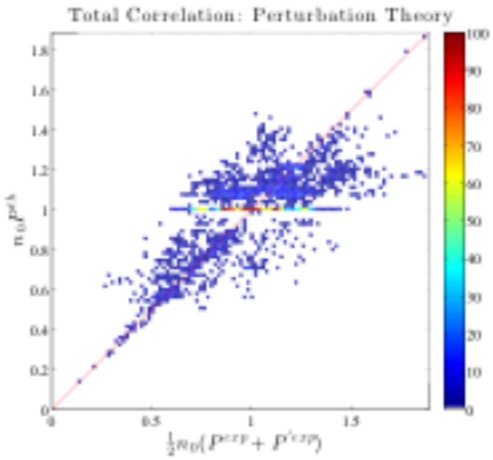
PRODUCT ANSATZ CORRELATIONS

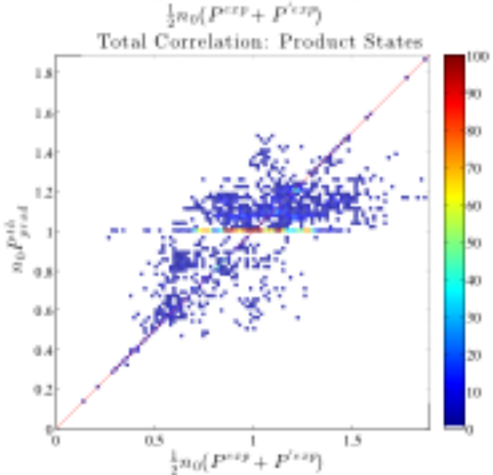
2x1 cell | 2x2 cell | 3x3 cell



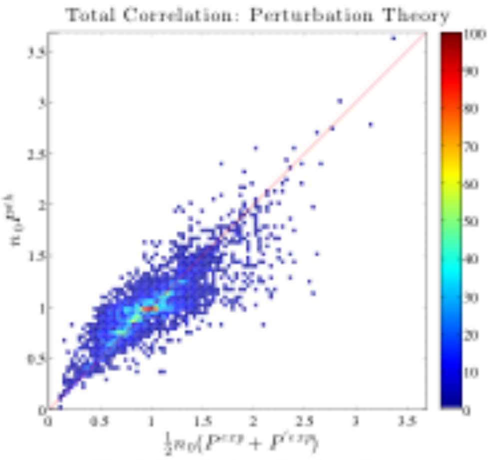
PRODUCT ANSATZ CORRELATIONS

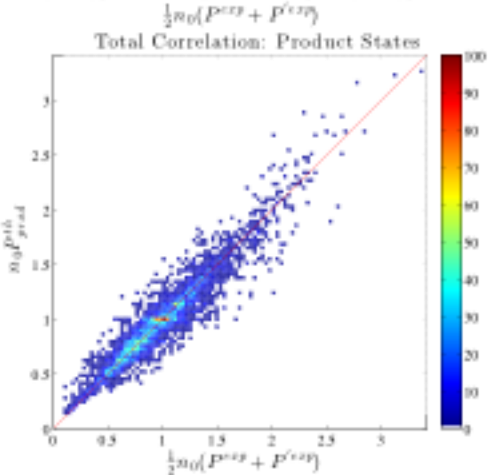
2x1 cell



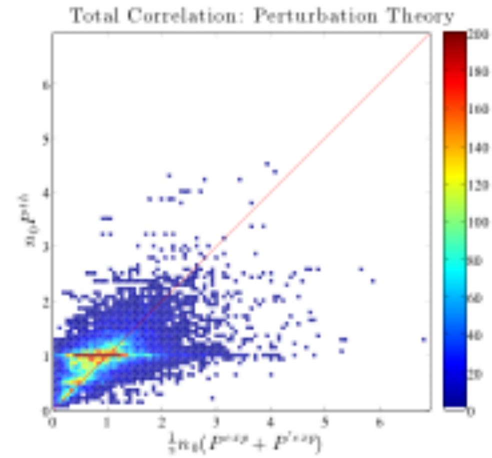


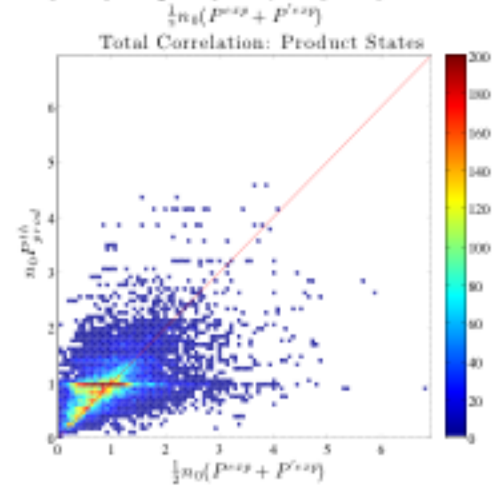
2x2 cell





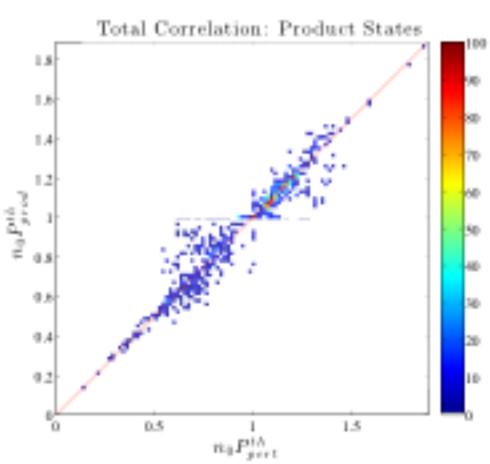
3x3 cell



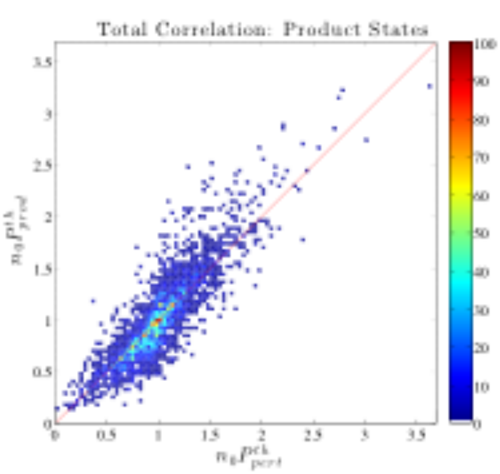


PRODUCT ANSATZ CORRELATIONS

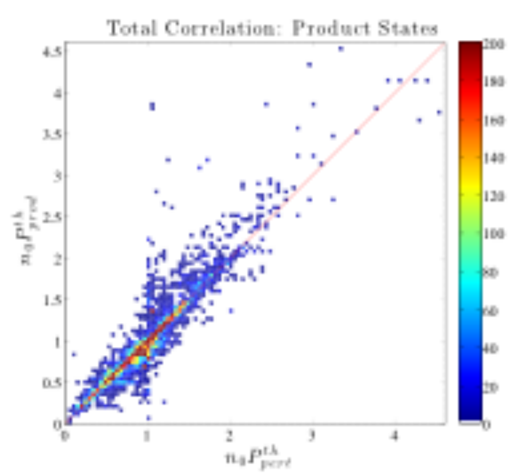
2x1 cell



2x2 cell

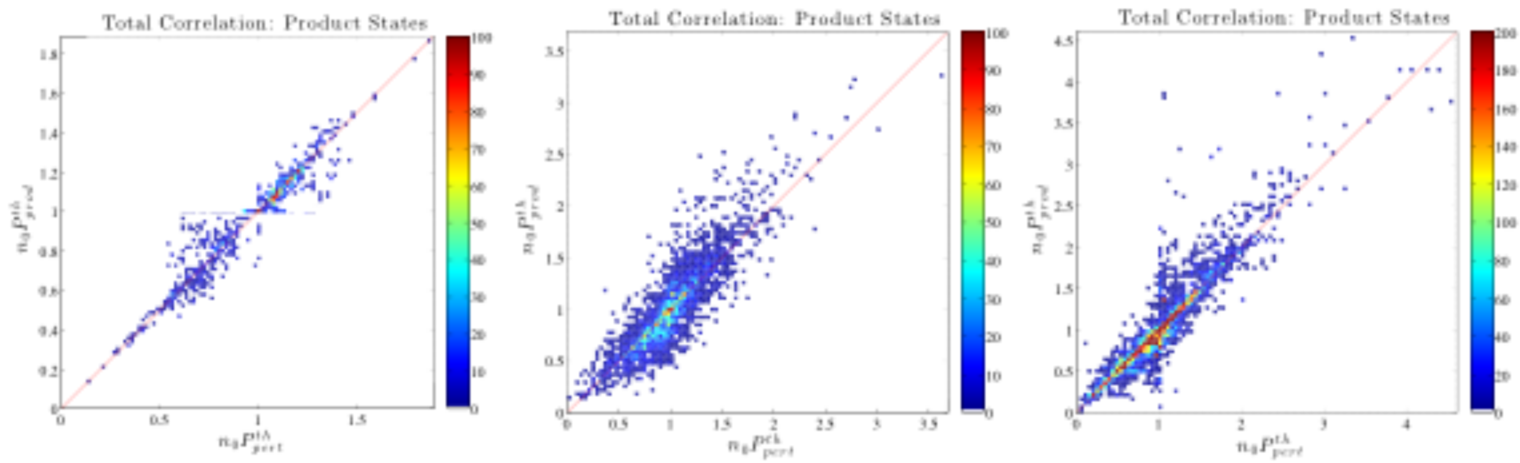


3x3 cell



PRODUCT ANSATZ CORRELATIONS

2x1 cell — 2x2 cell — 3x3 cell



- Entanglement is not required to reproduce the experimental signature on the set of instances considered

CLASSICAL SPIN DYNAMICS: SSSV MODEL

[Smith, Smolin, Vazirani *et al.* '13, '14]

- Monte Carlo simulations (Metropolis updates):
  - Product ansatz: $\Psi = \prod_i (\cos(\theta_i)|\uparrow\rangle_i + \sin(\theta_i)|\downarrow\rangle_i)$
  - Initial state: $\theta_i^0 = \pi/4$
  - Number of updates per spin: 6000

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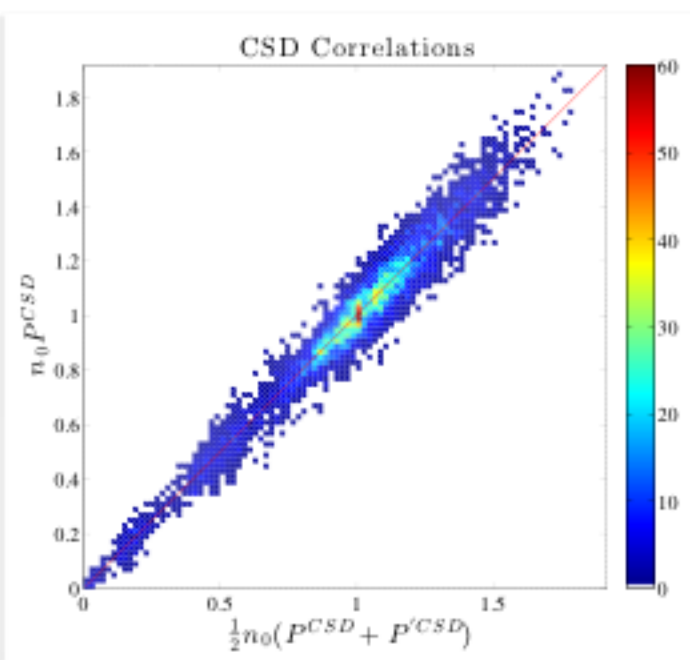
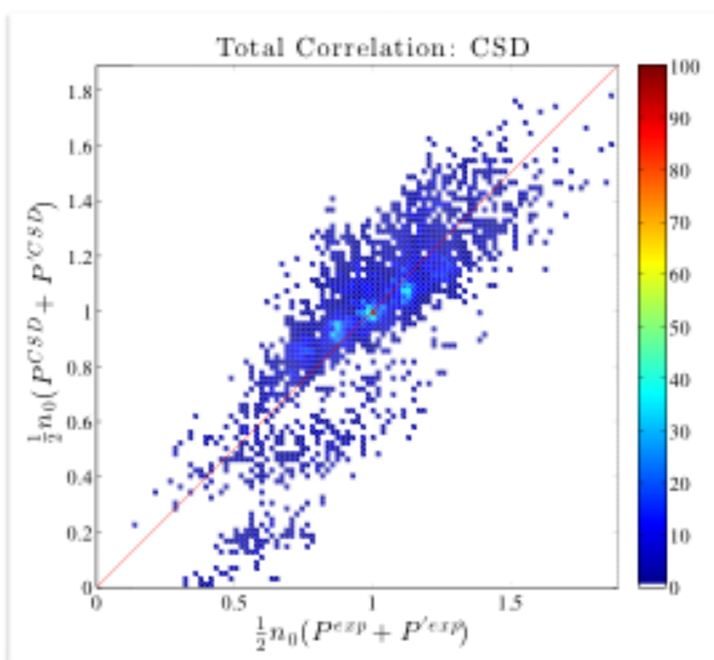
  - Number of updates per spin: 6000
  - Temperature: 17mK

SSSV CORRELATIONS: 2X1 CELL

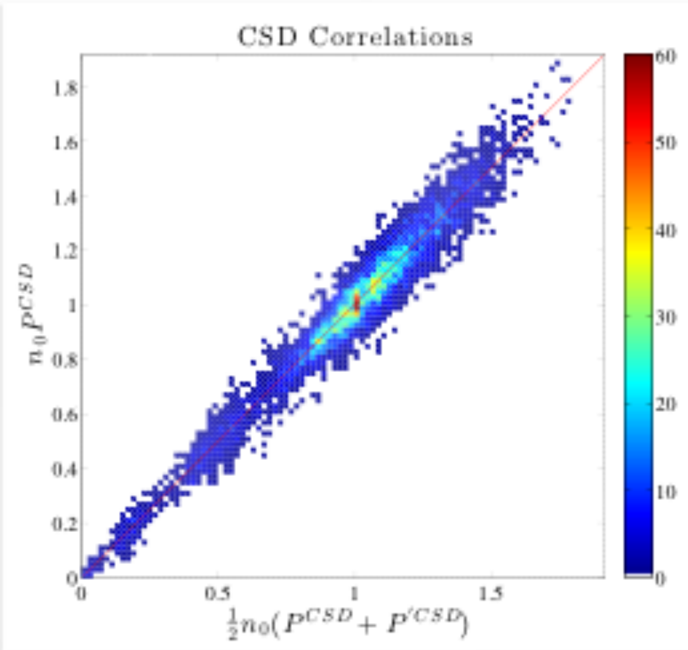
694 instances

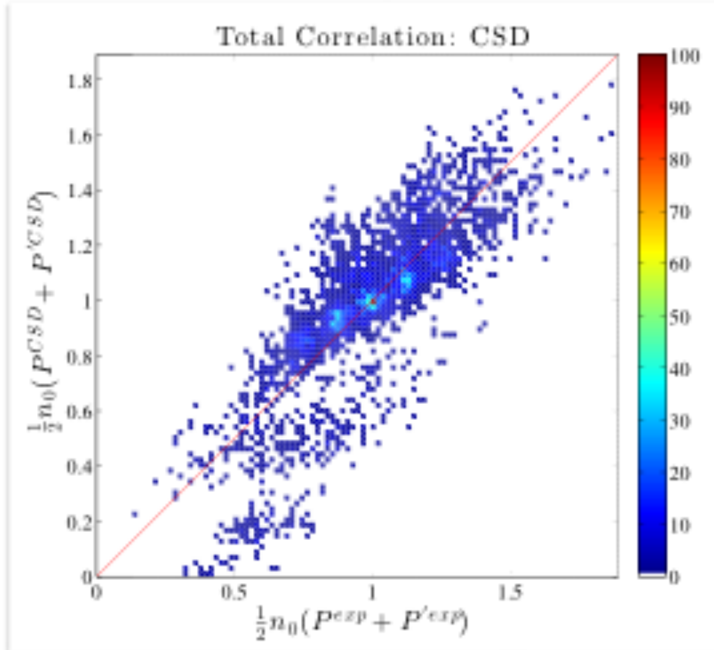




SSSV CORRELATIONS: 2X1 CELL

694 instances

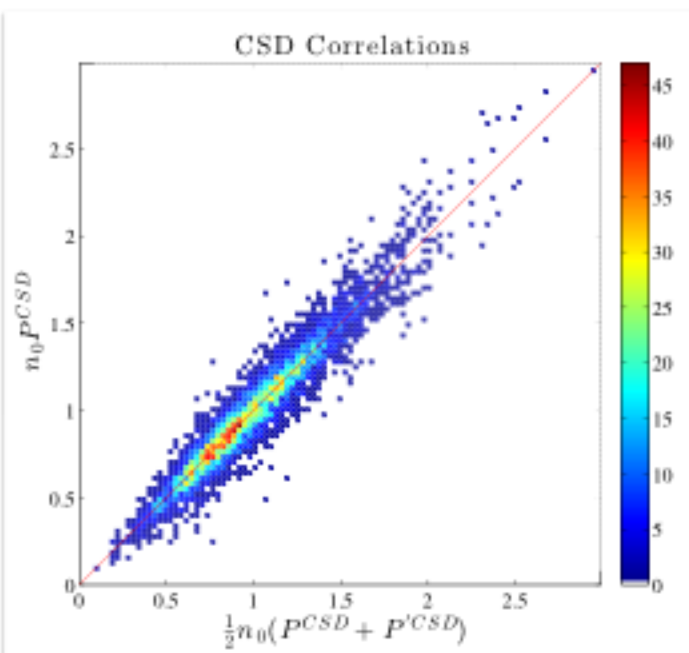


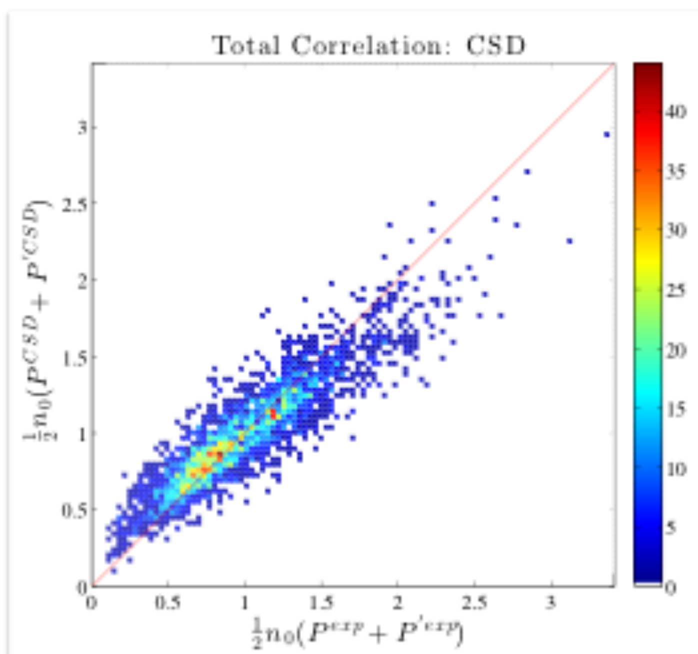


No instance-by-instance fine-tuning

SSSV CORRELATIONS: 2X2 CELL

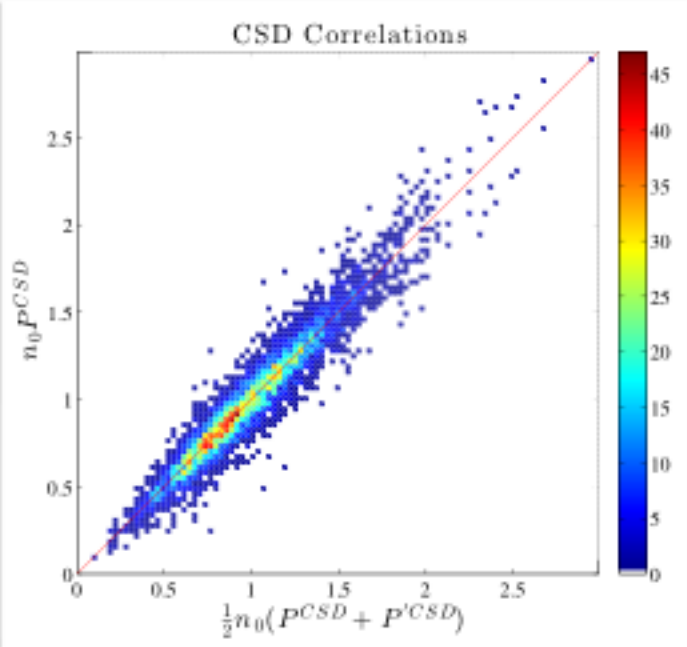
542 instances

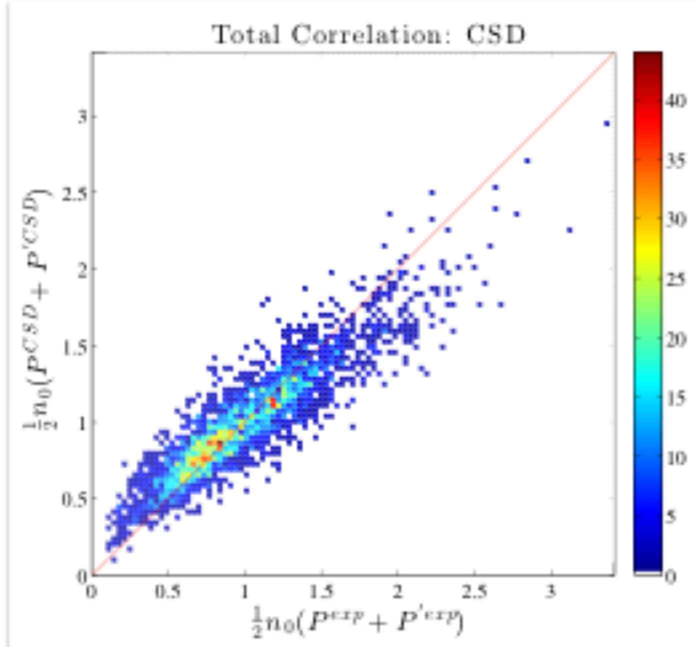




SSSV CORRELATIONS: 2X2 CELL

542 instances

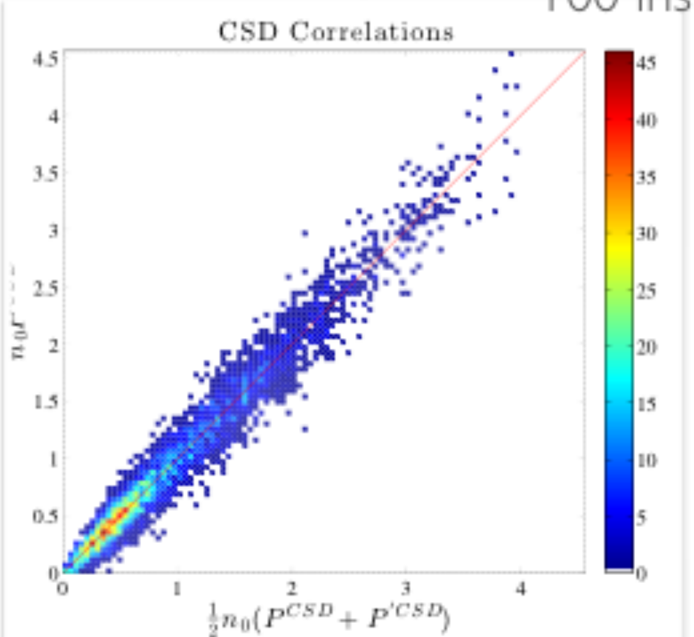


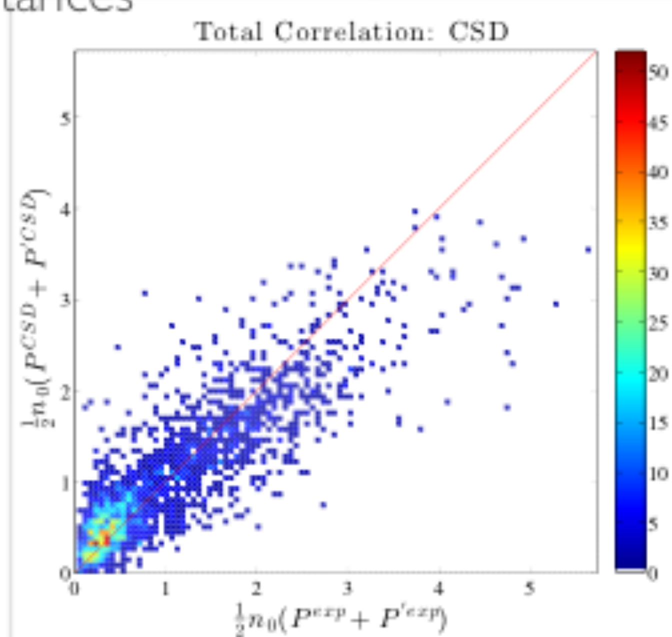


The SSSV model is effective in predicting
experimental suppression (enhancement)
for all the ground states of all the instances considered

SSSV CORRELATIONS: 3X3 CELL

100 instances

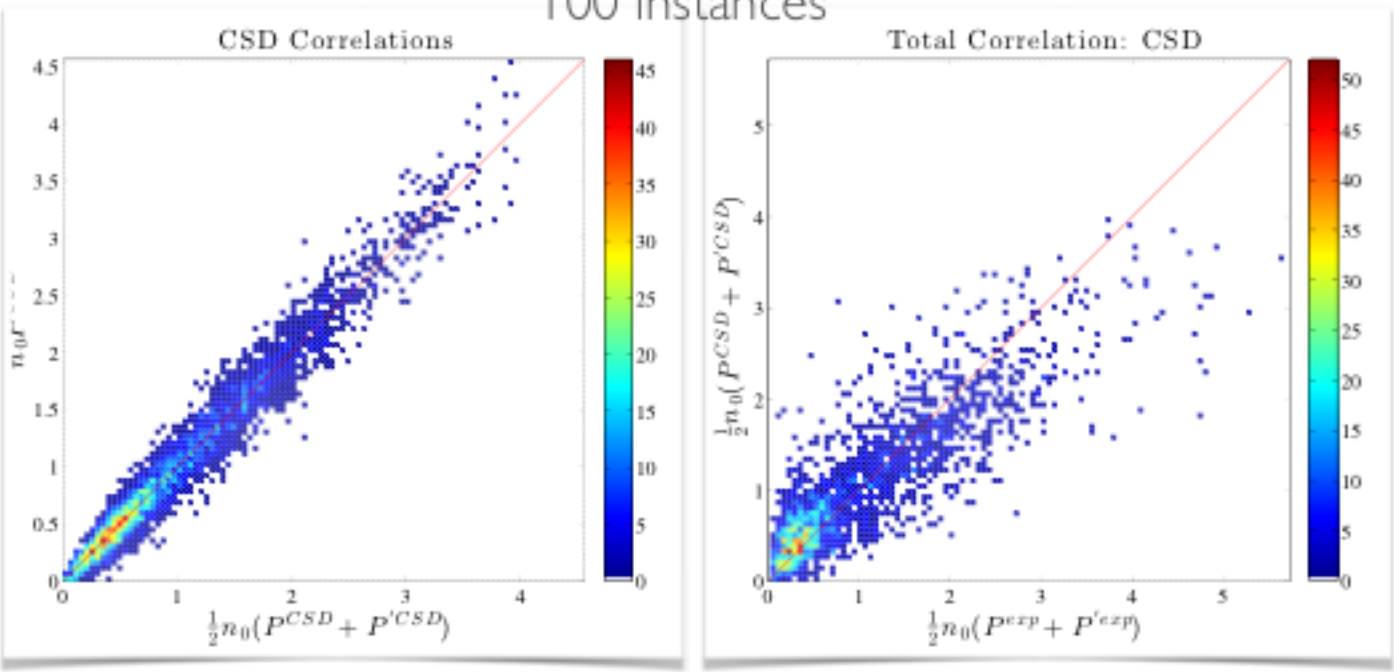




- The SSSV model is not perfectly accurate in reproducing the

SSSV CORRELATIONS: 5X5 CELL

100 instances



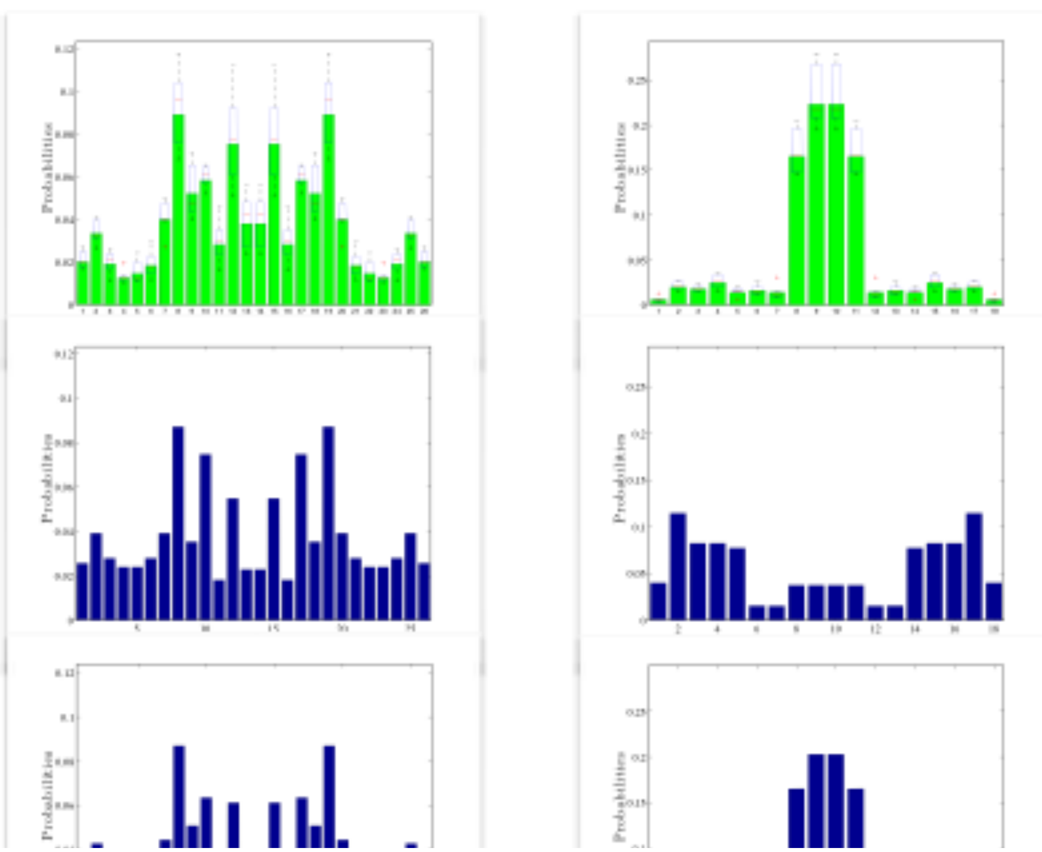
- The SSSV model is not perfectly accurate in reproducing the ground state signature
- Nonetheless, it didn't fail 'dramatically'

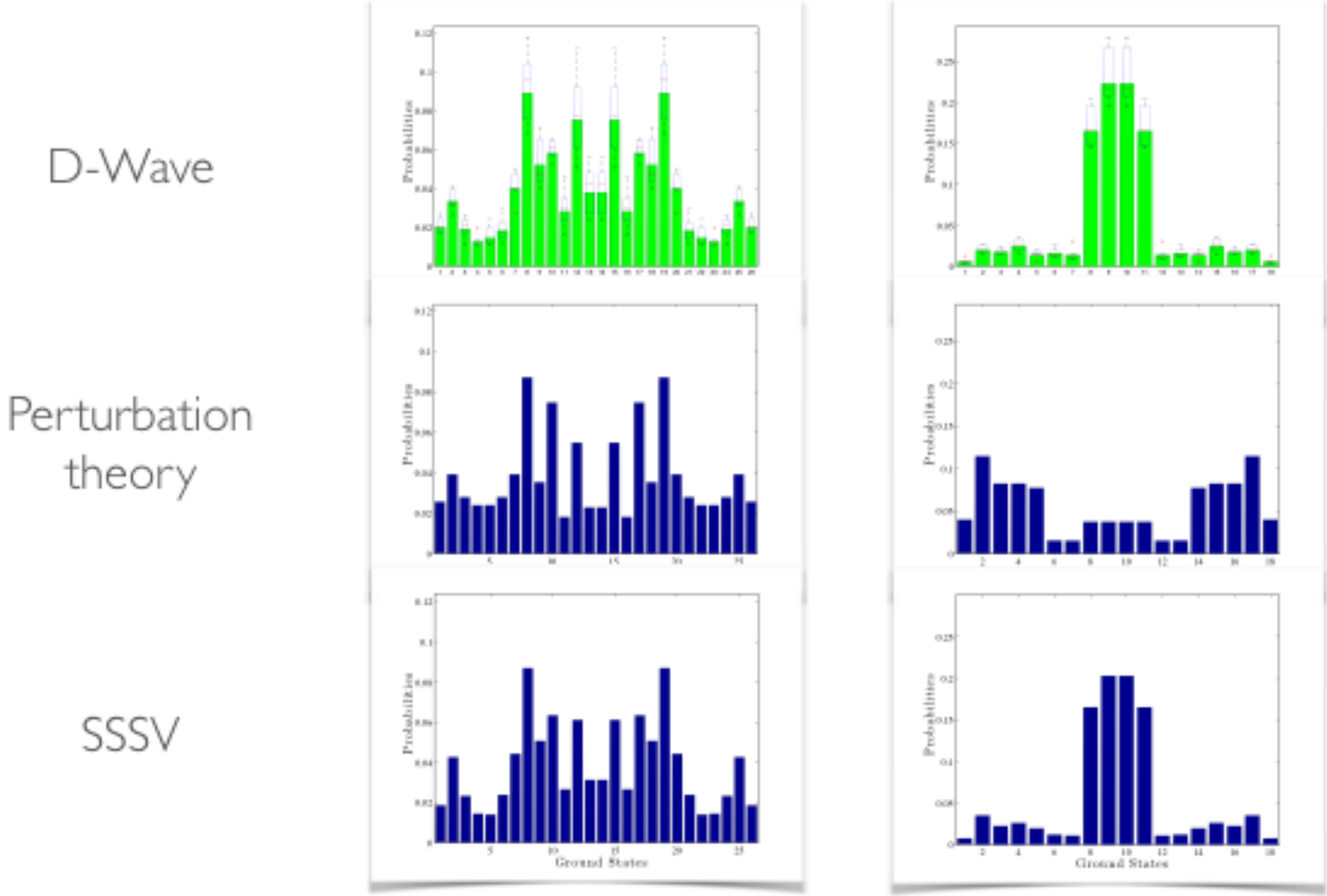
SSSV GROUND STATE EXAMPLES

D-Wave

Perturbation theory

SSSV





CONCLUSIONS

- The ground state signature for the D-Wave device can be reliably studied in a wide set of problem sizes
  - Correlates with the late-time evolution exact quantum spectrum (ideal quantum annealing)
  - Can be used as an indirect measurement of non-adiabaticity

- The ground state signature cannot be used (for the instances studied) as a test for quantum correlations
  - A product ansatz correlates as well
  - Classical spin dynamics (SSSV model) reproduces the main effects of non-adiabatic excitations in the non-perturbative

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ACKNOWLEDGMENTS

- Discussions:
  - Andrew Green, Nick Chancellor, Tanja Duric, Philip Crowley (LCN)
  - Daniel Lidar, Tameem Albash, Anurag Mishra (USC)

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LOCKHEED MARTIN