How Quantum is the annealing With The D-wave machine

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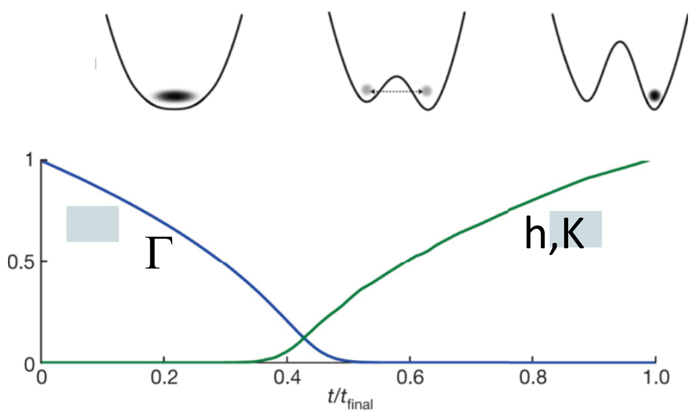
The D-Wave Qubits: design

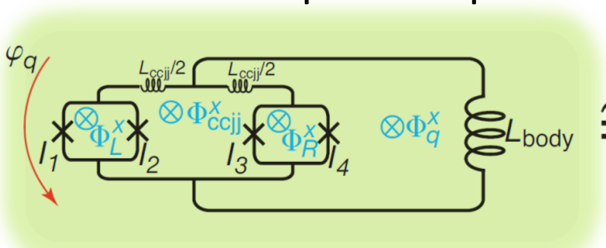
Annealing Schedule *(Johnson et al. 2012)*

$$\frac{\mathcal{H}_0(t)}{J_{\mathrm{AFM}}(t)} = -\sum_i h_i \sigma_z^{(i)} + \sum_{i,j>i} K_{ij}\sigma_z^{(i)}\sigma_z^{(j)} - \Gamma(t)\sum_i \sigma_x^{(i)}$$



- Qubit decoherence time ~ 20ns
- Typical interqubit coupling ~ 2GHz

- Actual RF-Squid compound Josephson Junction



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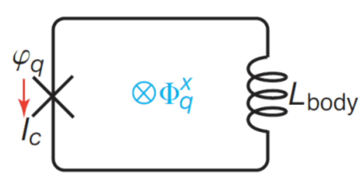
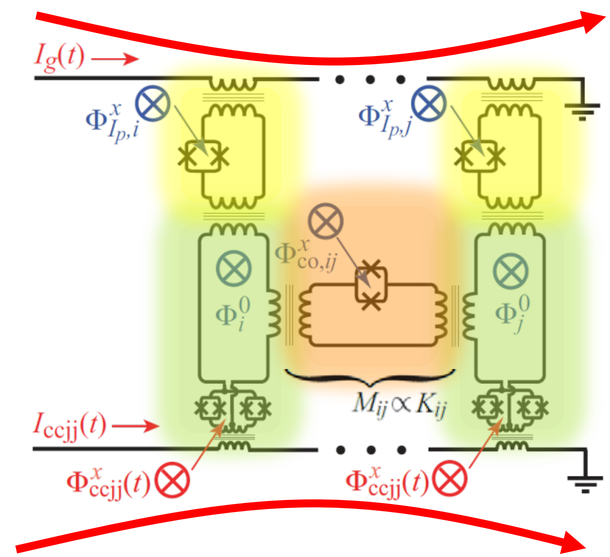


$I_c = \mathrm{Fcn}(\Phi_L^x, \Phi_R^x, \Phi_{\mathrm{ccjj}}^x)$

Scalable Qubit Architecture

(Johnson et al. 2010)



$$h_i = \frac{\Phi_i^x(t) - \Phi_i^0}{M_{\mathrm{AFM}}|I_q^p(t)|}$$

$$K_{ij} = \frac{M_{ij}(t)}{M_{\mathrm{AFM}}}$$

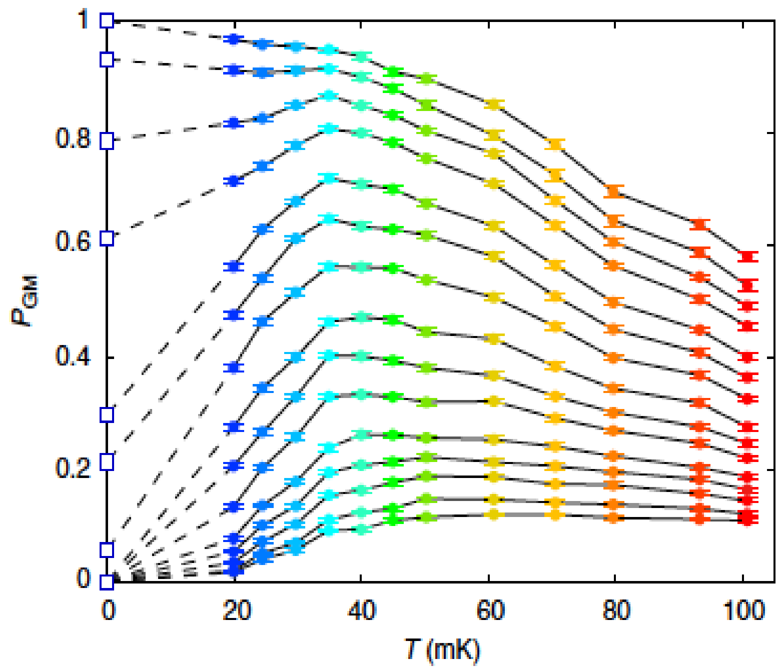
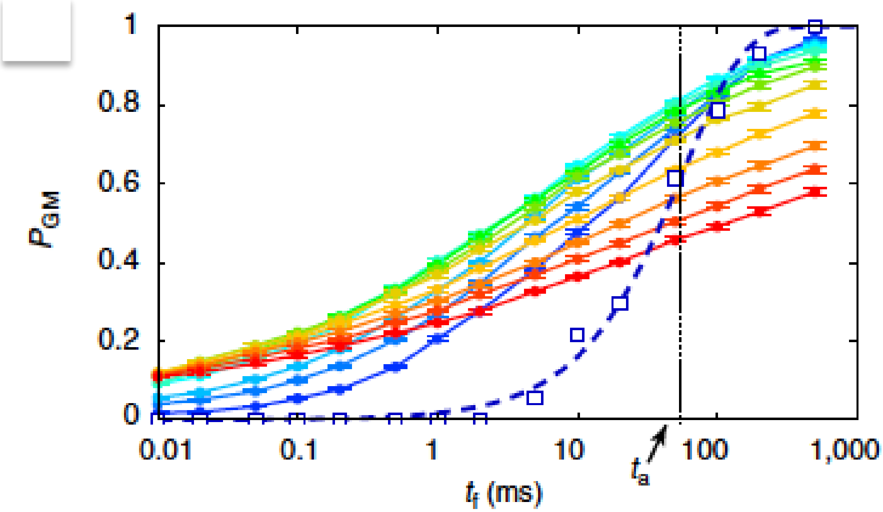


Figure 3 | Success probability for each anneal time as a function of temperature. Final ground-state probability as a function of temperature for, from top to bottom, $t_f = 500$, 200, 100, 50, 20, 10, 5, 2, 1, 0.5, 0.2, 0.1, 0.05, 0.02 and 0.01 ms. The measured temperatures are $T = 19.9$, 24.5, 29.7, 34.9, 40.0, 45.0, 50.3, 60.9, 70.6, 79.7, 93.3 and 100.8 mK. An initial enhancement of the performance is observed with increasing T up to $T_{peak} \sim 40$ mK, in agreement with the theoretical expectation depicted in Fig. 1. The blue squares are linear extrapolations of the curves to $T = 0$, subject to $0 \leq P_{GM} \leq 1$. The error bars depict twice the s.e.m., assuming each sample is independent.

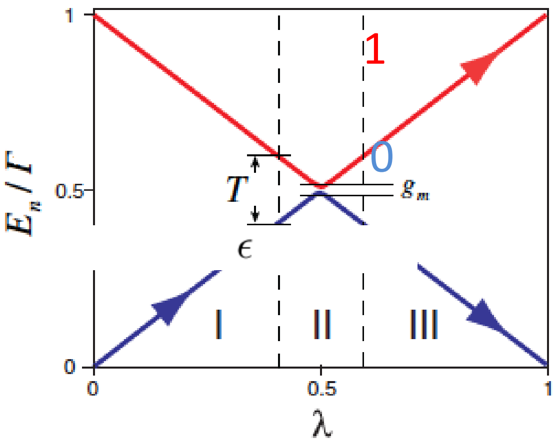


Success probability for each temperature as a function of anneal time.

The dashed blue lines are from fitting the expected closed-system behaviour to the extrapolated points, giving $t_a = 57.2$ ms.

$$t_a = 2\hbar v / \pi g_{min}^2$$

Resonant tunneling at Landau-Zeener transition



$$H_S = -(\epsilon\sigma_z + g_m\sigma_x)/2, \qquad H_{\rm int} = -Q\sigma_z/2,$$

$$\dot{\rho}_z = -\Gamma(\rho_z - \rho_\infty)$$

$$\Gamma = \Gamma_{01} + \Gamma_{10} \text{ and } \rho_\infty = [\Gamma_{10} - \Gamma_{01}]/\Gamma$$

$$\Gamma_{01}(\epsilon) = \frac{g_m^2}{4}\int dt e^{i\epsilon t} \exp\left\{\int \frac{d\omega}{2\pi} S(\omega) \frac{e^{-i\omega t} - 1}{\omega^2}\right\} \qquad S(\omega) = \int_{-\infty}^{\infty} dt\; e^{i\omega t}\langle Q(t)Q(0)\rangle$$

$$p_G = 1 - e^{-\gamma t_f},$$
$$\gamma \equiv \frac{1}{t_f}\int_0^{\infty} \Gamma(\epsilon)\frac{d\epsilon}{\dot{\epsilon}} = \frac{1}{t_f}\int_{-\infty}^{\infty} \Gamma_{01}(\epsilon)\frac{d\epsilon}{\dot{\epsilon}}$$

Resonant tunneling

$$\dot{\epsilon} = \mathrm{const} \equiv \nu,$$

$$\gamma t_f = \frac{1}{\nu}\int_{-\infty}^{\infty} \Gamma_{01}(\epsilon)d\epsilon = \pi g_m^2/2\nu$$

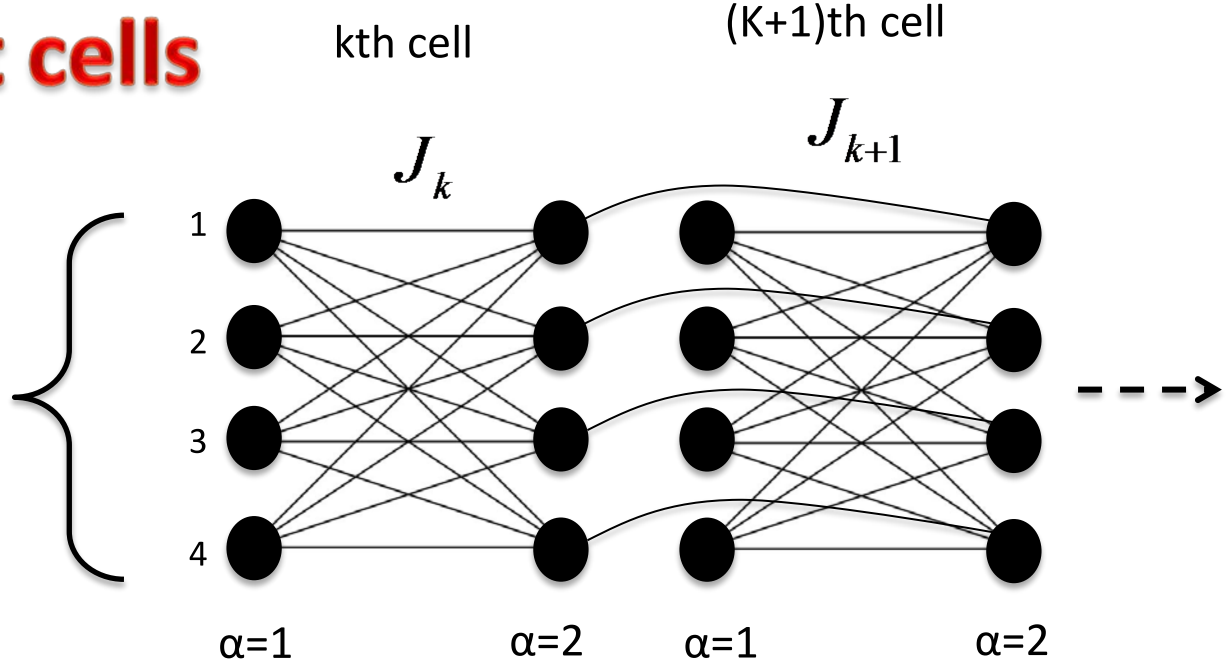
$$\Gamma_{01}(\epsilon) = \frac{g_m^2}{4}\int dt e^{i\epsilon t} \exp\left\{\int \frac{d\omega}{2\pi} S(\omega) \frac{e^{-i\omega t}-1}{\omega^2}\right\}$$

At low T LZ probability is unaffected by coherence. The physical reason for this is that the decoherence changes only the profile of the transition region while keeping the total transition probability the same.

P. Ao and J. Rammer, Phys. Rev. B 43, 5397 (1991)
Y. Kayanuma and H. Nakayama, Phys. Rev. B 57, 13099 (1998)

Chimera-type graph chain of unit cells

Unit cell has 2 columns and n rows



$$H_P = \sum_k H_P^k + H_P^{k,k+1}$$

$$H_P^k = -J_k \sum_{i \neq j=1}^{n} \sigma_{i,z}^{1,k} \sigma_{j,z}^{2,k} - \sum_{\alpha=1,2} \sum_{i=1}^{n} h_{\alpha,k}\, \sigma_{i,z}^{\alpha,k}$$

$$H_P^{k,k+1} = -\sum_{i=1}^{n} J_i^{k,k+1} \sigma_{i,z}^{1,k} \sigma_{i,z}^{1,k+1}$$

- Strong coupling inside clusters
- Weak coupling between clusters

$$\sum_{i=1}^{n} |J_i^{k,k+1}| \ll |J_k|$$

Each column corresponds to a spin n/2

$$S_z^{1,k} = \frac{1}{2}\sum_{j=1}^{n} \sigma_{j,z}^{1,k}, \qquad S_z^{2,k} = \frac{1}{2}\sum_{j=1}^{n} \sigma_{j,z}^{2,k}$$

$$H_P^k = -4J_k\, S_z^{1,k}\, S_z^{2,k} - 2\sum_{\alpha=1,2} h_{\alpha,k} S_z^{\alpha,k}$$

$$H_P^{k,k+1} = -\bar{J}_{k,k+1}\, S_z^{2,k}\, S_z^{2,k}$$

$$\boxed{\bar{J}_{k,k+1} = \frac{1}{n}\sum_{i=1}^{n} J_i^{k,k+1}}$$

Effective coupling between cells

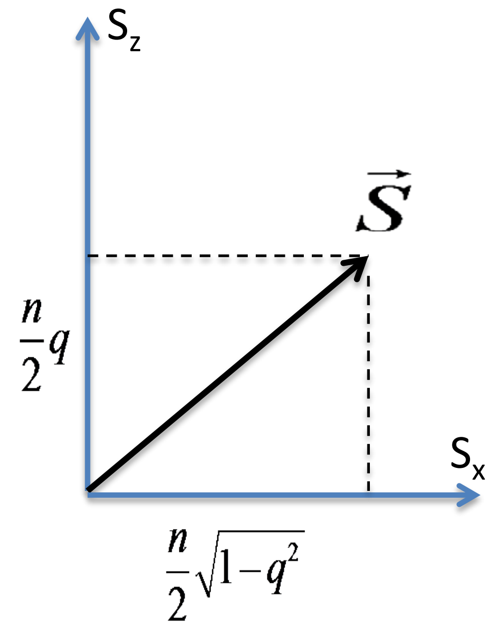
Wentzel-Kramers-Brillouin method for large spins n>>1

$$S_z = \frac{n}{2} q, \quad S_x = \frac{n}{2}\sqrt{1-q^2}\cos\hat{p} + \mathcal{O}(1), \qquad p = -i\,\frac{2}{n}\,\frac{d}{dq}$$

Villain transformation

$$S_+ = \frac{n}{2} e^{-ip}\,\left[1 + 2/n - q(q+2/n)\right]^{1/2}$$

$$S_x = \frac{1}{2}\left(S_+ + S_-\right)$$



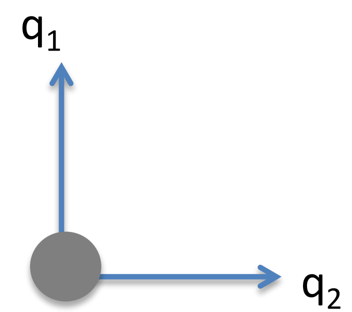
$$H_{WKB}(\mathbf{q}, \hat{\mathbf{p}}, s) = -(1-s)n \sum_{\alpha=1,2} \sum_{k} \sqrt{1 - q_{k,\alpha}^2} \cos p_{k,\alpha}$$

$$-ns\left(\frac{n^2}{4} J_k q_{k,1} q_{k,2} + h_{k,1} q_{k,1} + h_{k,2} q_{k,2} \right)$$

$$-ns \bar{J}_{k,k+1}\, q_{1,k}\, q_{1,k+1}$$

$$H_{WKB}(\mathbf{q}, \hat{\mathbf{p}}, s) = -n + \sum_{k} \sum_{\alpha=1,2} \frac{p_{k,\alpha}^2 + q_{k,\alpha}^2}{2}$$

Ground state wavefunction
Is Gaussian centered at
q1=q2=0



Ground state wave-function is Gaussian centered at the point

corresponds to Minimum of the potential

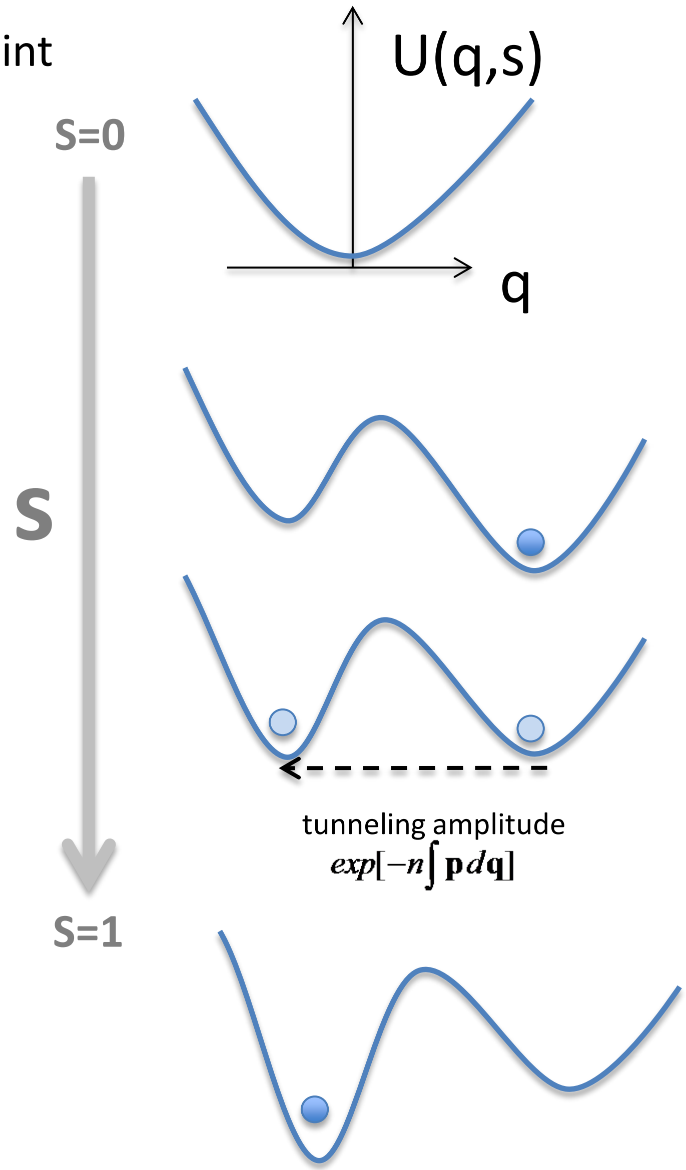
$$U(\mathbf{q}, s) = H_{WKB}(\mathbf{q}, \mathbf{p} = \mathbf{0}, s)$$

$$S_z = \frac{n}{2} q, \quad S_x = \frac{n}{2}\sqrt{1 - q^2}$$

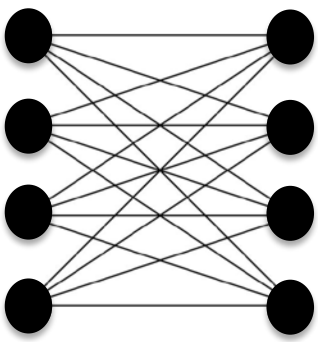
Wave-package follows minimum of the potential
as QA parameter s changing from 0 to 1

In case of global bifurcation minima changing order and
System just tunnel to succeed in QA

$$p(q,s) = \cosh^{-1}\left(\frac{s}{1-s}\frac{U(q,s)}{\sqrt{1-q^2}}\right)$$

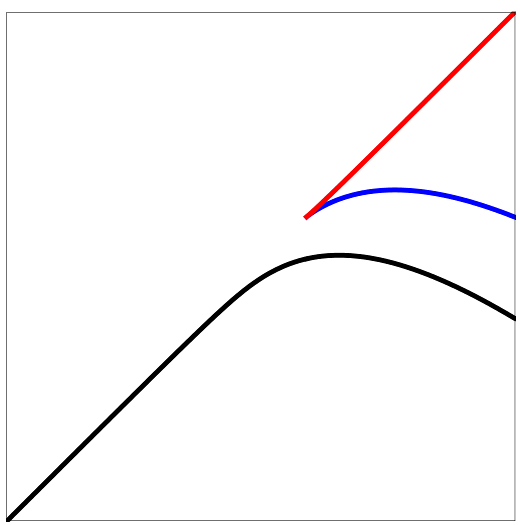


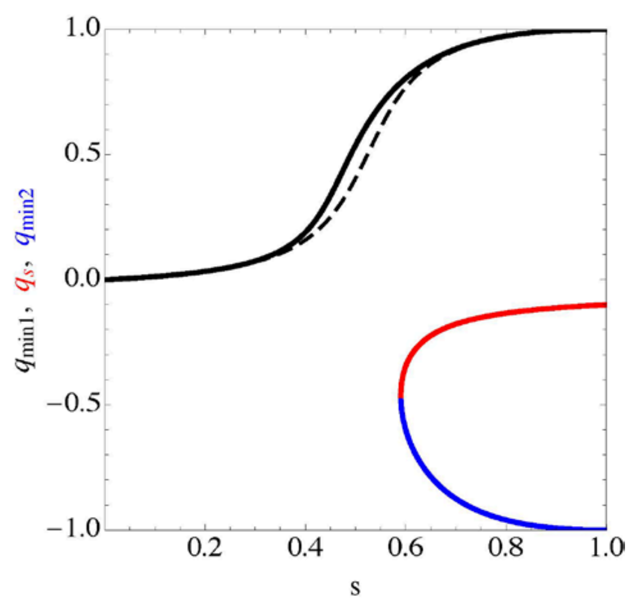
Bifurcation diagrams



- Small Bias z-field h identical in all spins
- Ferromagnetic coupling J equal for all spins
- h << J cost function has 2 closely spaced minima

q_1 and q_2 corresponds to spins of left and right columns

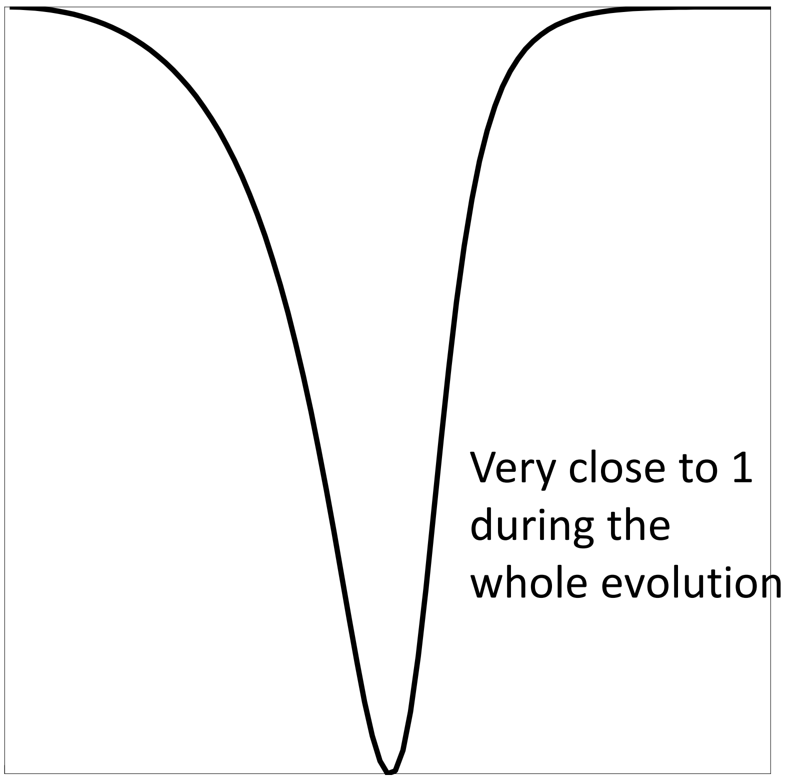


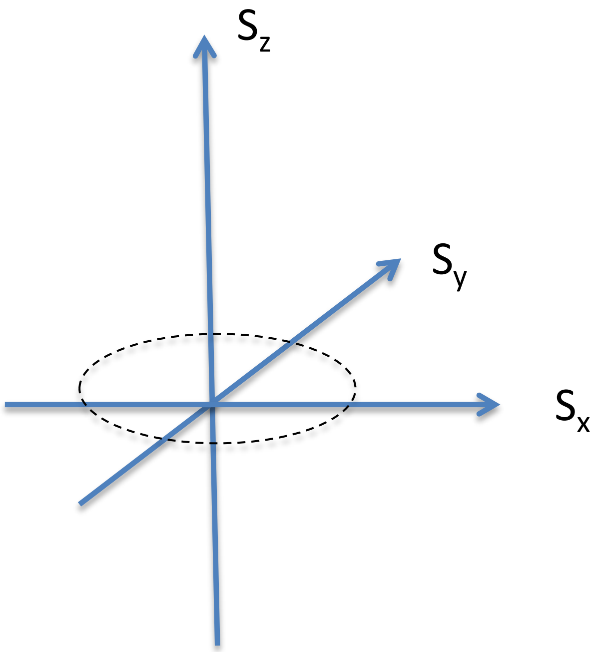


Global minimum evolves smoothly without any bifurcations during QA

$$\frac{2}{n}\left(\langle S_x\rangle^2+\langle S_y\rangle^2+\langle S_z\rangle^2\right)$$

$$\Psi(m_z)=\Psi(-m_z)$$





Avoidance of tunneling leads to purely classical evolution

Quantum Adiabatic Evolution Algorithms With Different Paths

- E. Farhi, J. Goldstone, and S. Gutmann, arXiv:quant-ph/0208135.
- Yuya Seki, Hidetoshi Nishimori, arXiv:1203.2418 [quant-ph]
- Victor Bapst, Guilhem Semerjian arXiv:1203.6003 [cond-mat.stat-mech]

Mechanism to form a global bifurcation

System does not know about the inter-cell coupling early in QA when s<< 1

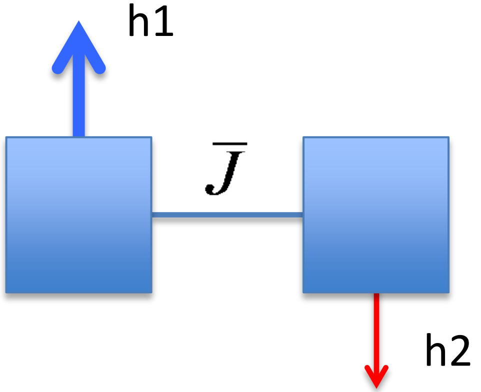
z-polarizations are small

$$H \approx -(1-s)n\left(1-\frac{1}{2}q^2-\frac{1}{8}q^4-\ldots\right)-ns\,\bar{J}_0 q_1 q_2 - ns\; h(q_1+q_2)$$

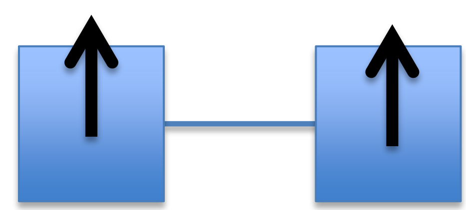
$$q_1 \approx q_2 \approx s\; h$$

Inter-cell coupling is next order in smallness compare to local fields

$$\bar{J}_{k,k+1}\, q_1 q_2 \sim s^2\; h_1 h_2$$



$$\boxed{h_1 - h_2 < J/2} \Longrightarrow$$

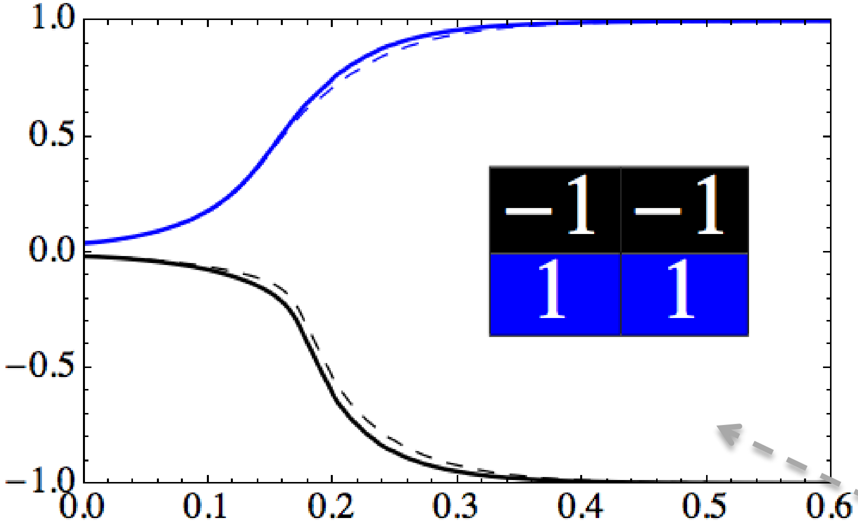


Ferromagnetic ground state
Spins aligned in the same direction

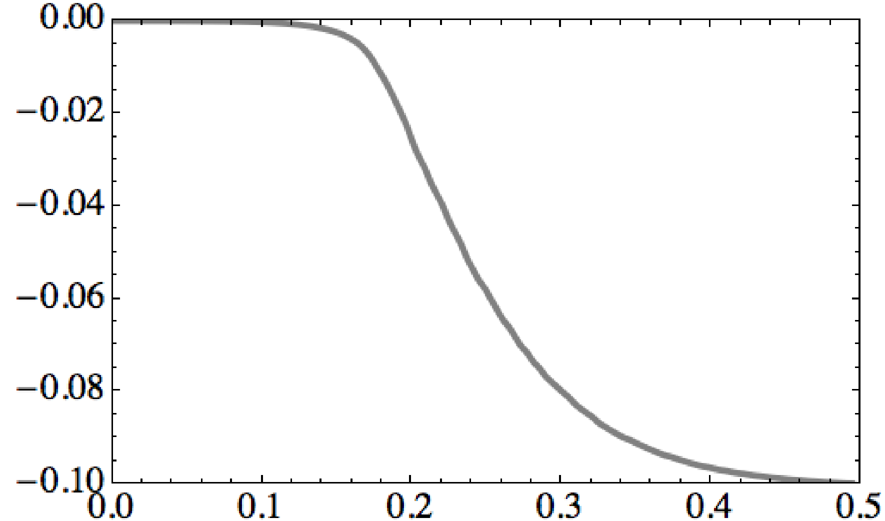
Two-cluster problem

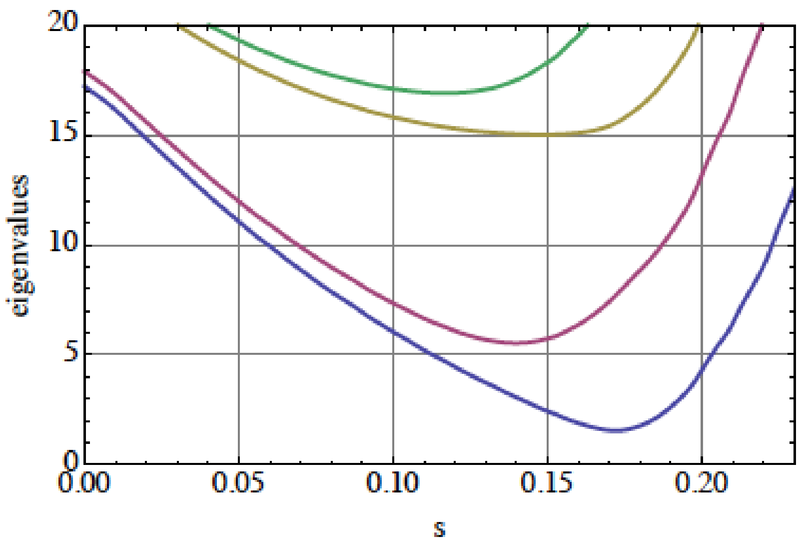
h1=-0.46 **h2=1** J=1

$E_1 - E_0 = 2.09069 \qquad e_{\min 2} = 1.54811$



Fracion of inter−cluster energy





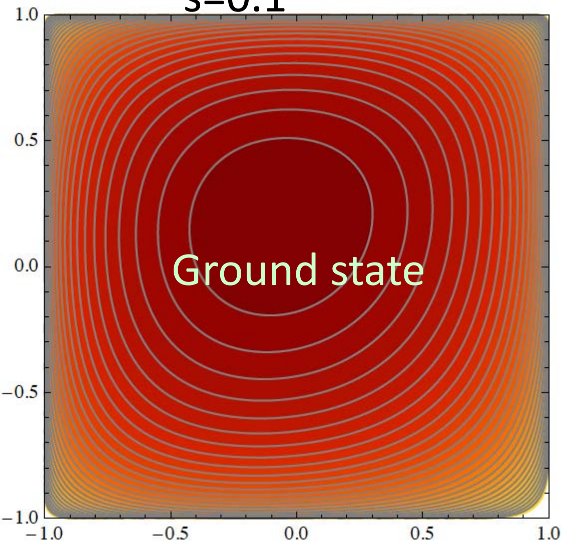
Clusters begin to move in opposite directions

Classical solution is ferromagnetic

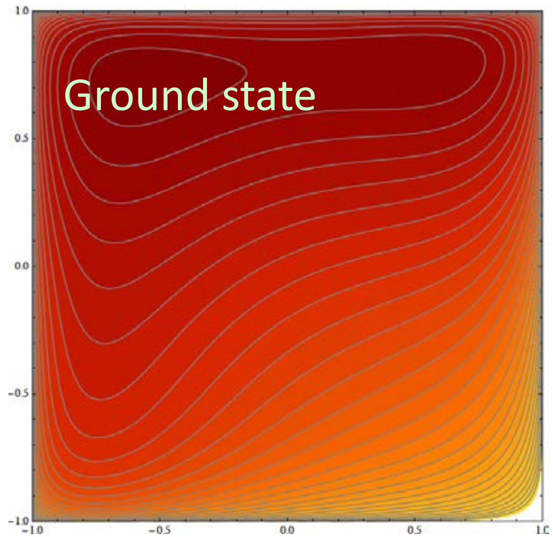
-131.713	-129.623	-103.489
$\boxed{\begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array}}$	$\boxed{\begin{array}{cc} -1 & -1 \\ 1 & 1 \end{array}}$	$\boxed{\begin{array}{cc} -1 & -1 \\ -1 & -1 \end{array}}$

Two-cluster problem : successive density plots (q1,q2)

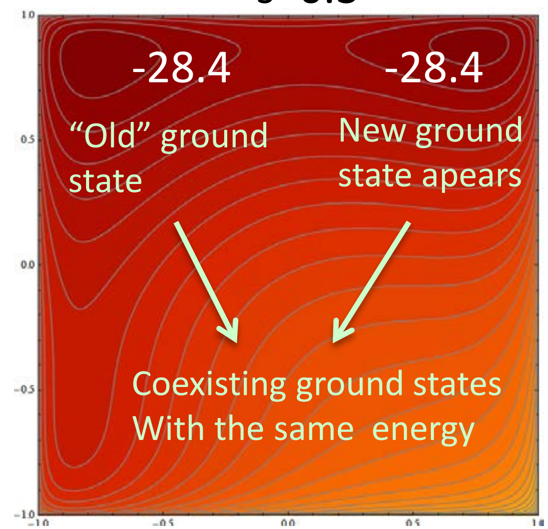
s=0.1



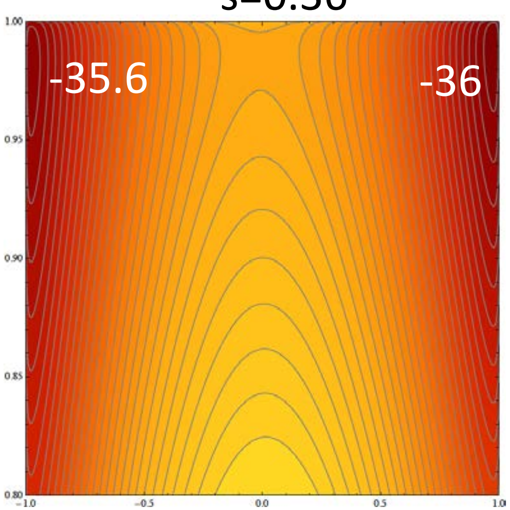
s=0.2

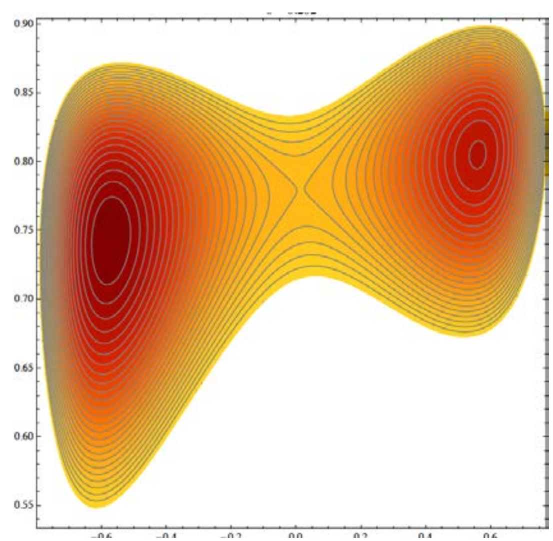


s=0.3



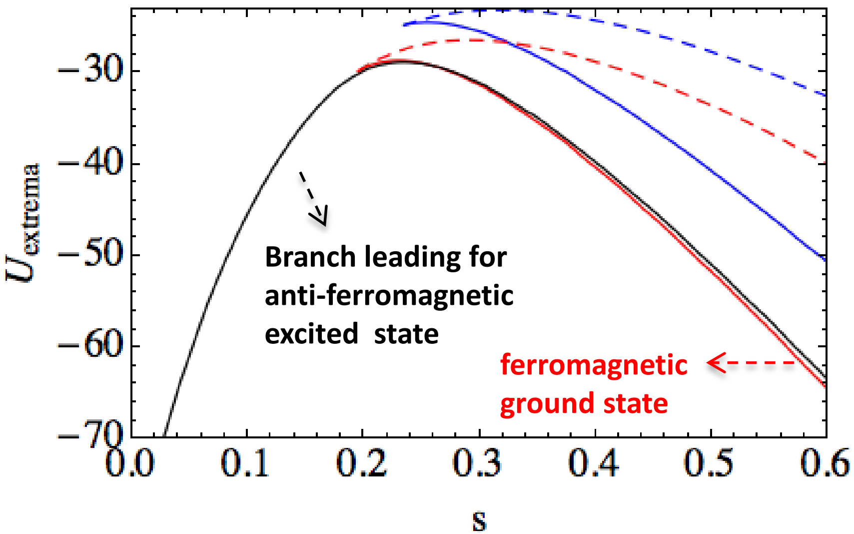
s=0.36



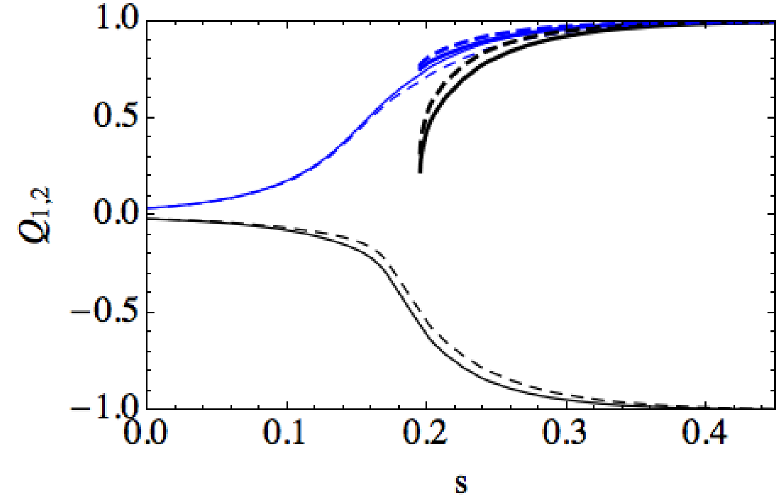


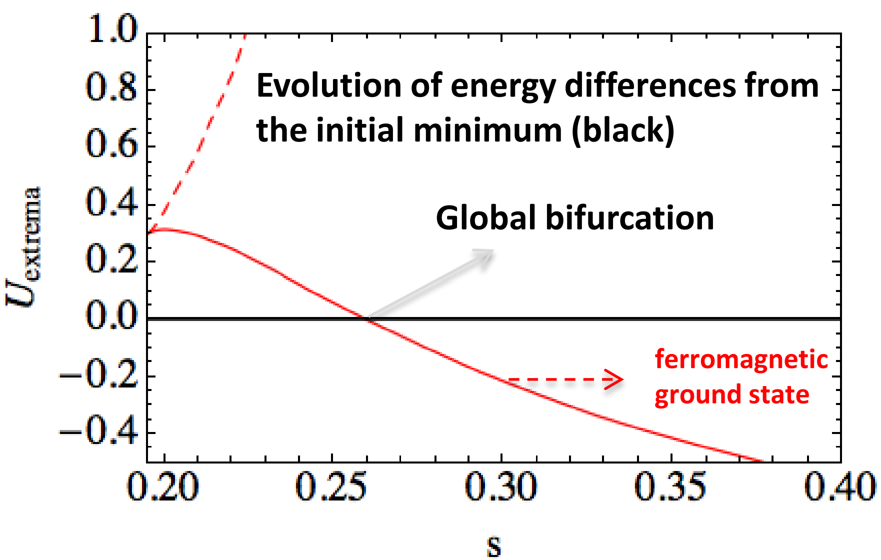
-131.713	-129.623	-103.489
$\boxed{\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}}$	$\boxed{\begin{matrix} -1 & -1 \\ 1 & 1 \end{matrix}}$	$\boxed{\begin{matrix} -1 & -1 \\ -1 & -1 \end{matrix}}$

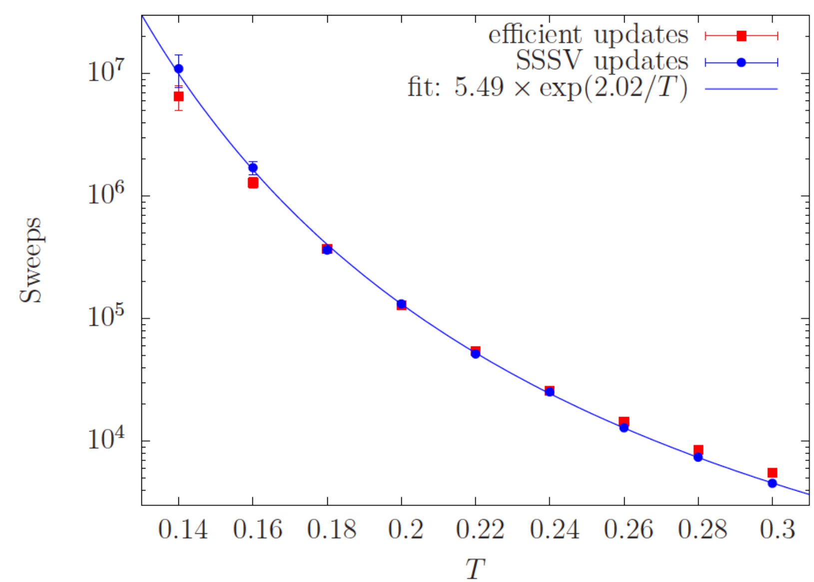
Evolution of energies
Of local minima and saddle points



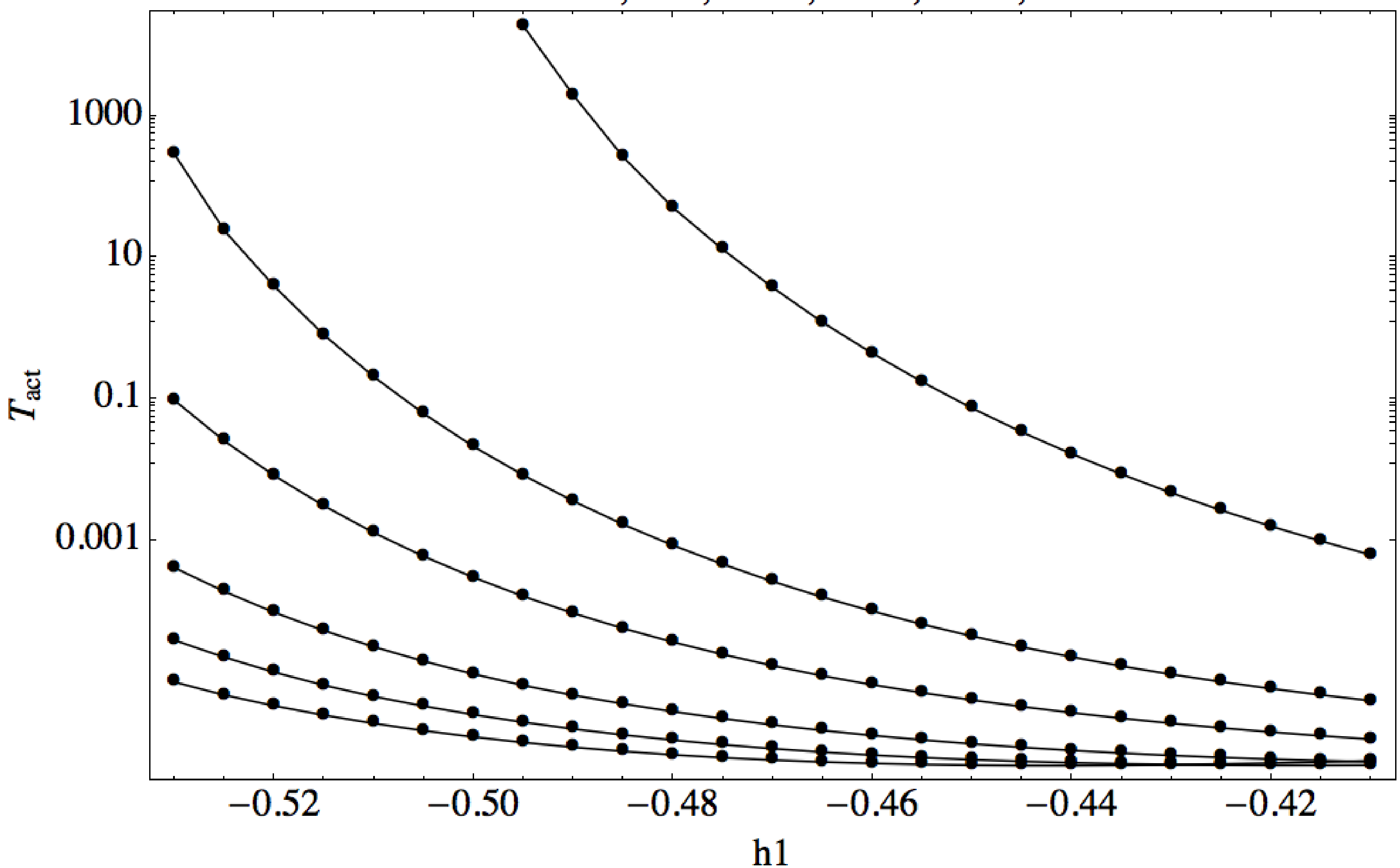
Evolution of "coordinates"
of local minima

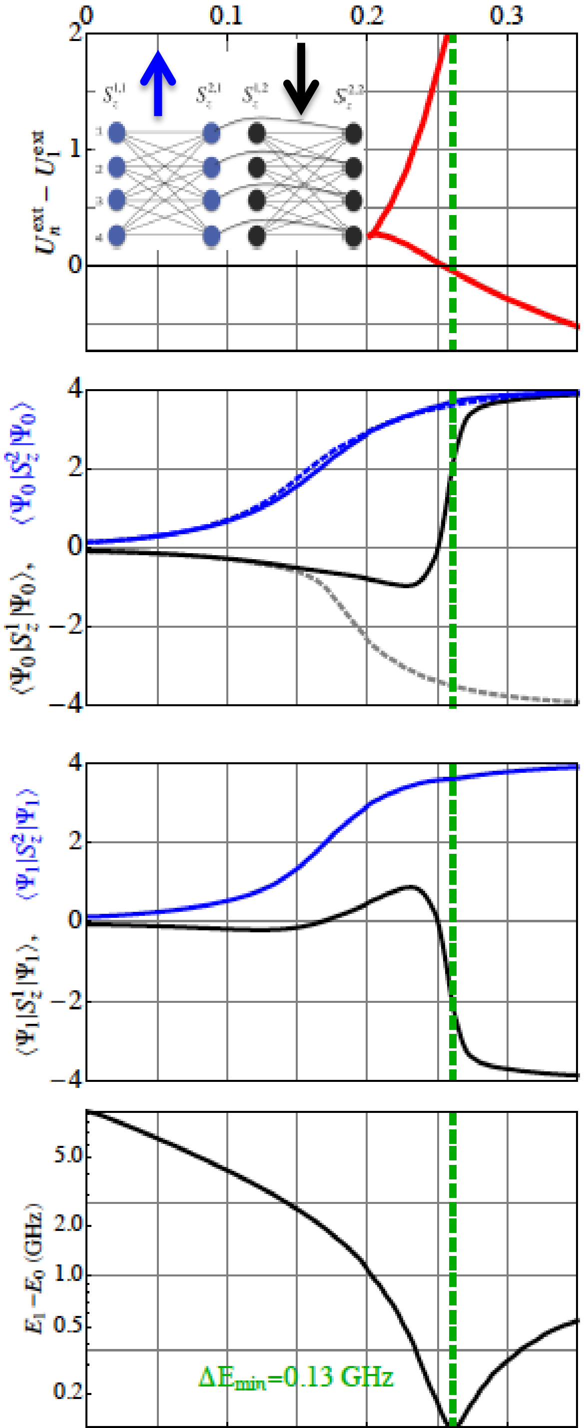






error= 1%, 5%, 10%, 20%, 30%, 40%





National Aeronautics and Space Administration
0
0.1
0.2
0.3
$U_n^{ext} - U_1^{ext}$
$\langle\Psi_0|S_z^1|\Psi_0\rangle$, $\langle\Psi_0|S_z^2|\Psi_0\rangle$
$\langle\Psi_1|S_z^1|\Psi_1\rangle$, $\langle\Psi_1|S_z^2|\Psi_1\rangle$
$E_1 - E_0$ (GHz)
ΔE_{min}=0.13 GHz

Ohmic (High Frequency) Noise

$$H_{\mathrm{int}} = \sum_{i=1}^{N} \sigma_i^z Q_i \qquad S(\omega) = \int_{-\infty}^{-\infty} dt \langle Q(t)Q(0)\rangle = \hbar J(\omega)[1+\bar{n}(\omega)] = S_{LF} + S_{HF}$$

$$S_{\mathrm{HF}}(\omega) = \hbar^2 J_{\mathrm{HF}}(\omega)/(1 - e^{-\beta\hbar\omega})$$

$$J(\omega) = \eta\omega|\omega/\omega_c|^s e^{-|\omega|/\omega_c}$$

Ohmic Coefficient

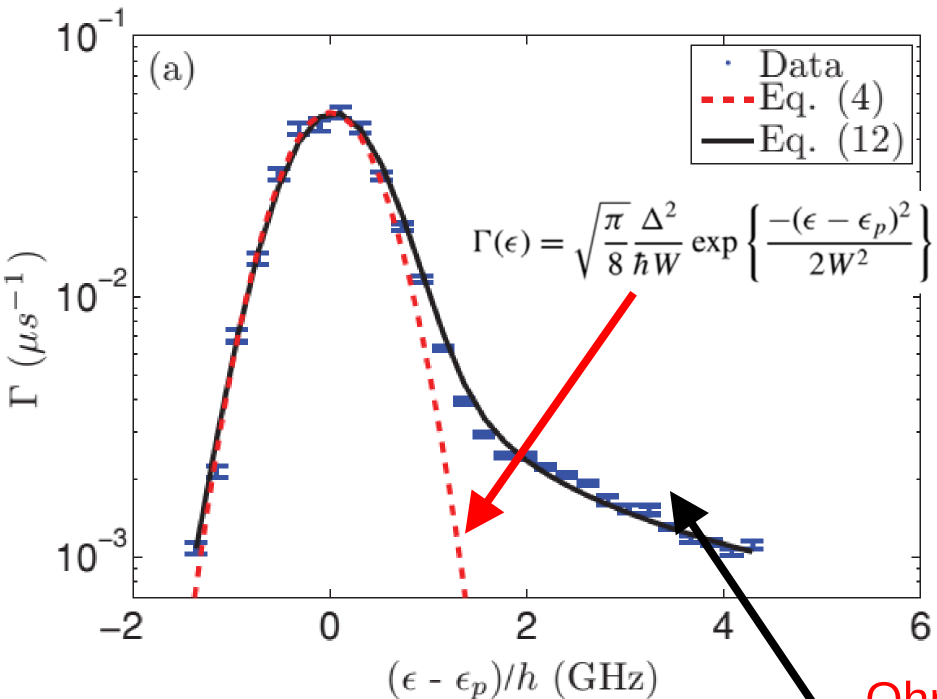
$$\Gamma(\epsilon) = \frac{\Delta_r^2}{4\hbar} \int d\tau e^{i(\epsilon-\epsilon_p)\tau - W^2\tau^2/2} \left[i \sinh \frac{\tau - i\tau_c}{\beta/\pi} \right]^{-\frac{2\eta}{\pi}}$$

$$\Delta_r = (\pi/\beta\omega_c)^{\frac{\eta}{\pi}} \Delta$$

(Lanting et al., PRB 83, 180502(R) (2011))

Probing High Frequency Noise

(Lanting et al., PRB 83, 180502(R) (2011))



Ohmic shoulder

Noise parameters	2011
$\vert$	0.1
W	470 GHz
ε_p	350MHz

$$\Gamma(\epsilon) = \frac{\Delta_r^2}{4\hbar}\int d\tau e^{i(\epsilon-\epsilon_p)\tau - W^2\tau^2/2}\left[i \sinh\frac{\tau - i\tau_c}{\beta/\pi}\right]^{\frac{-\eta}{2\pi}}$$

$$\Gamma(\epsilon) = \frac{\Delta_r^2}{4\hbar} \int d\tau \, e^{i(\epsilon - \bar{\epsilon}_p)\tau - \bar{W}^2 \tau^2/2} \left[i \sinh \frac{\tau - i\tau_c}{\beta/\pi} \right]^{\frac{-\bar{\eta}}{2\pi}}$$

$$\bar{\varepsilon}_p = a\,\varepsilon_p \qquad \varepsilon_p = \int \frac{d\omega}{2\pi} \frac{S_{LF}(\omega)}{\omega} = \frac{W^2}{k_B T}$$

$$\bar{W} = a W \qquad W = \int \frac{d\omega}{2\pi} S_{LF}(\omega)$$

$$\bar{\eta} = a\,\eta \qquad \mathrm{J} = \eta\omega \exp(-\omega/\omega_c)$$

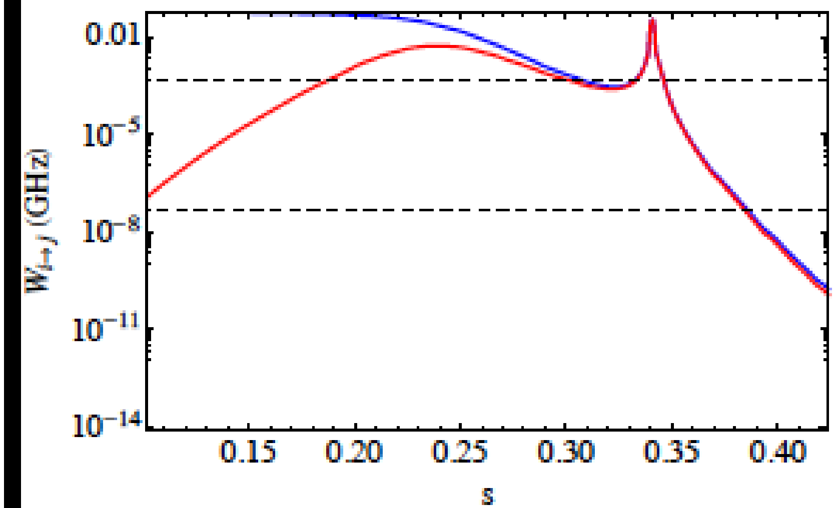
$$a = \frac{1}{4} \sum_n (\sigma_z^{n,22} - \sigma_z^{n,11})^2$$

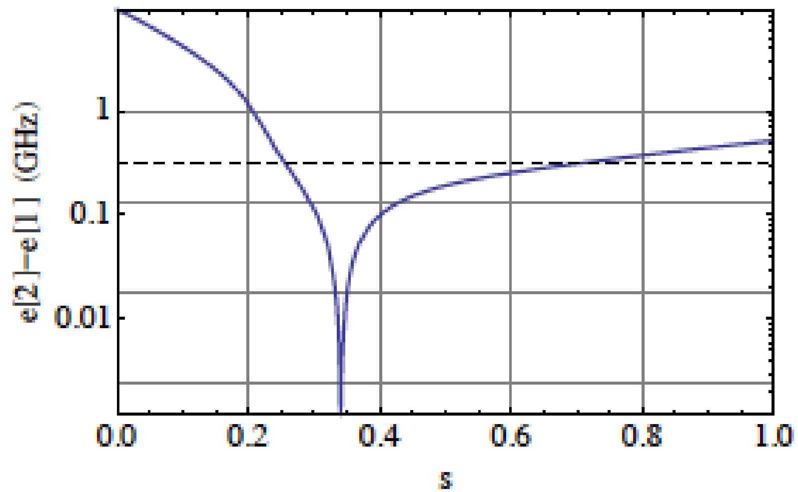
h1=−0.49 s_{10}^{min}=0.341 Δ_{10}^{min}=0.00132029 GHz Δ_{20}^{min}=2.83486 GHz T= 0.314 GHz

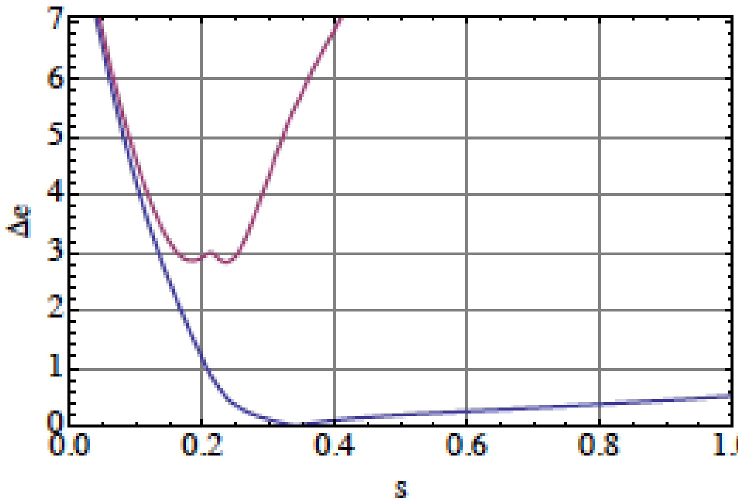
$s1_{freez}$ = 0.332888 $s2_{freez}$ = 0.385252 $(W[2,1]/\Delta e)[s1_{freez}]$ =2.82844e−2 $(W[2,1]/\Delta e)[s2_{freez}]$ =6.33904e−7

$W[s_{min}]/\Delta_{min}$= 24.9451 Δ_{min}/QAυfast= 2.64058 $\Delta e[s2_{freez}]$= 0.0176776 $\Delta e[s2_{freez}]$ =0.0788763

$p_2(s1_{freez})$= 0.485919 $p_2(s2_{freez})$ = 0.437482





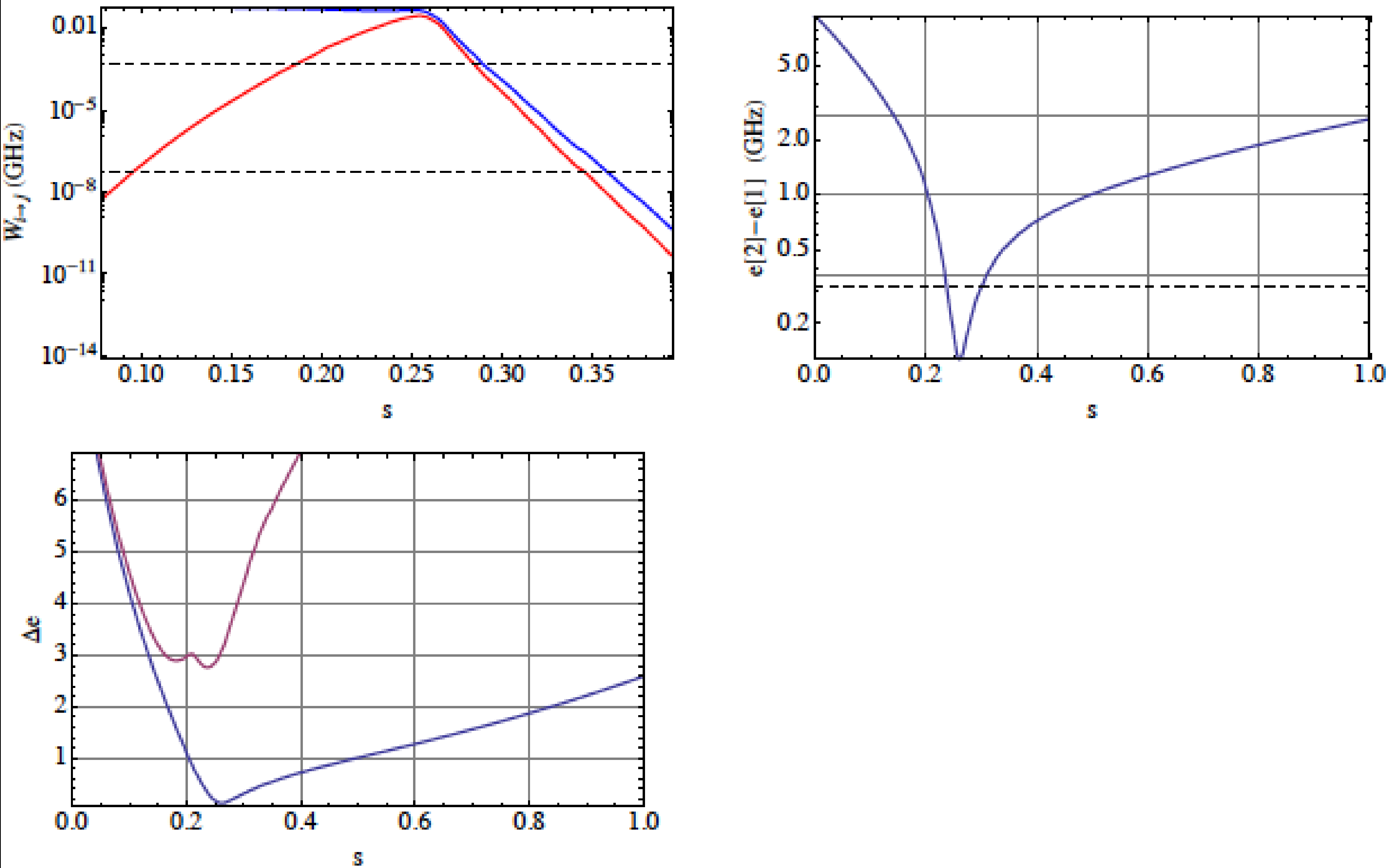


h1=−0.45 s_{10}^{min}=0.261 Δ_{10}^{min}=0.127622 GHz Δ_{20}^{min}=2.76251 GHz T= 0.314 GHz

$s1_{freez}$ = 0.289191 $s2_{freez}$ = 0.358732 $(W[2,1]/\Delta e)[s1_{freez}]$ =1.91958e−3 $(W[2,1]/\Delta e)[s2_{freez}]$ =8.63955e−8

$W[s_{min}]/\Delta_{min}$= 0.246746 Δ_{min}/QAιfast= 255.243 $\Delta e[s2_{freez}]$= 0.260474 $\Delta e[s2_{freez}]$ =0.578734

$p_2(s1_{freez})$= 0.303611 $p_2(s2_{freez})$ = 0.13652

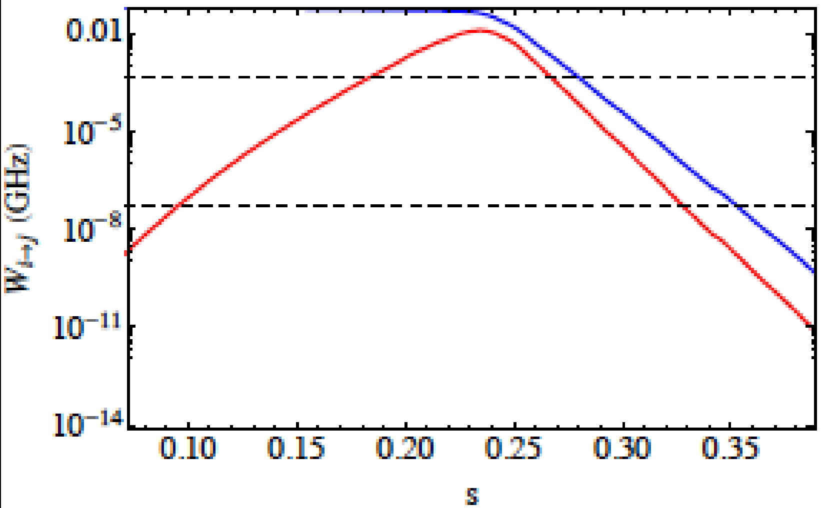


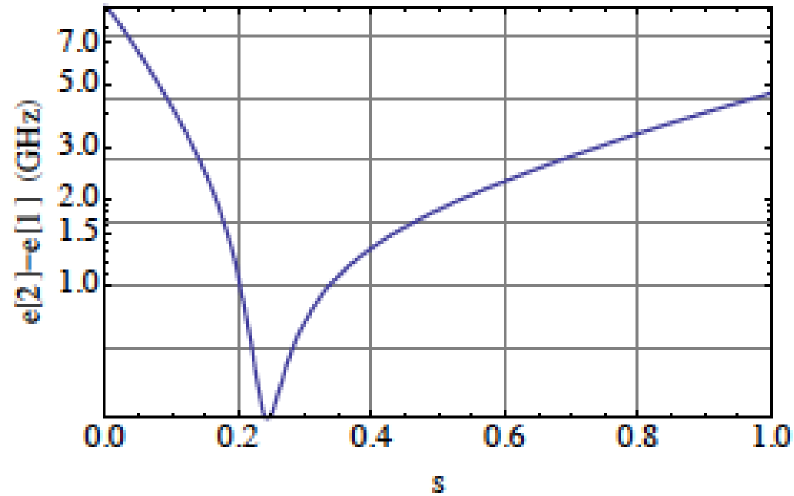
$h1=-0.41$ $s_{10}^{min}=0.243$ $\Delta_{10}^{min}=0.342234$ GHz $\Delta_{20}^{min}=2.77279$ GHz $T= 0.314$ GHz

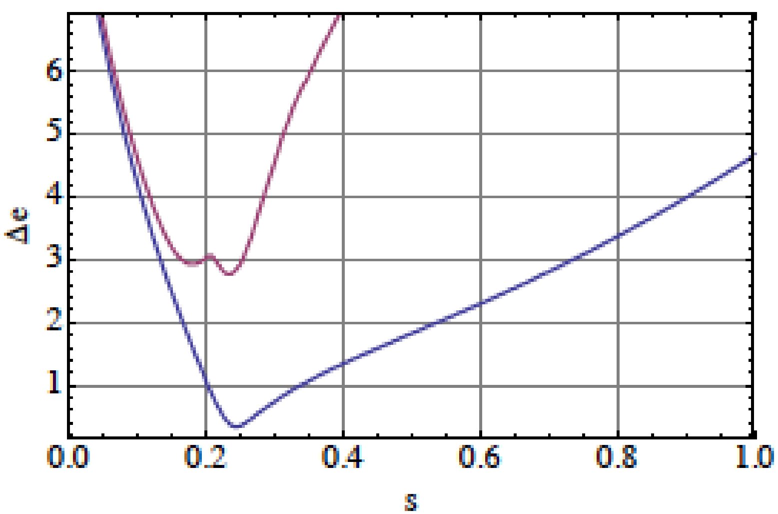
$s1_{freez} = 0.278812$ $s2_{freez} = 0.353024$ $(W[2,1]/\Delta e)[s1_{freez}] = 8.65156e-4$ $(W[2,1]/\Delta e)[s2_{freez}] = 4.59337e-8$

$W[s_{min}]/\Delta_{min}= 0.078007$ $\Delta_{min}/QA\upsilon fast= 684.469$ $\Delta e[s2_{freez}]= 0.57793$ $\Delta e[s2_{freez}] = 1.08853$

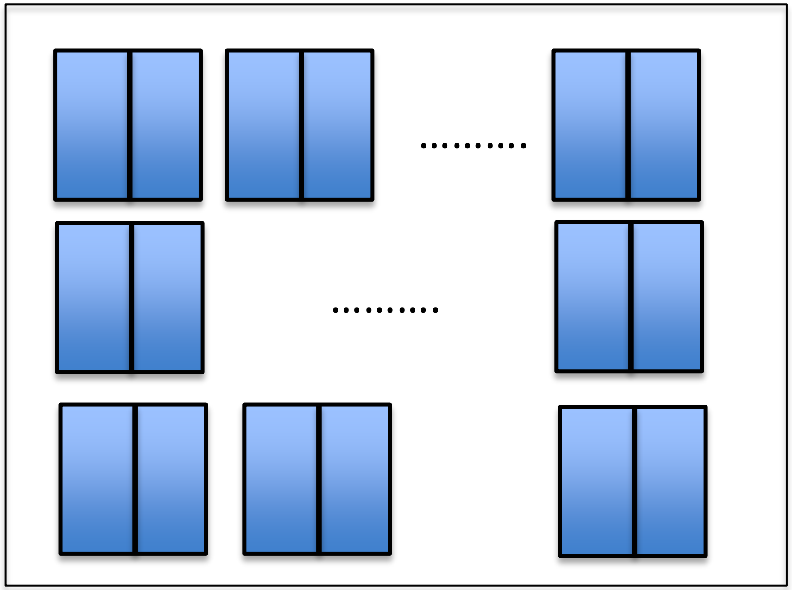
$p_2(s1_{freez})= 0.136823$ $p_2(s2_{freez}) = 0.0301994$







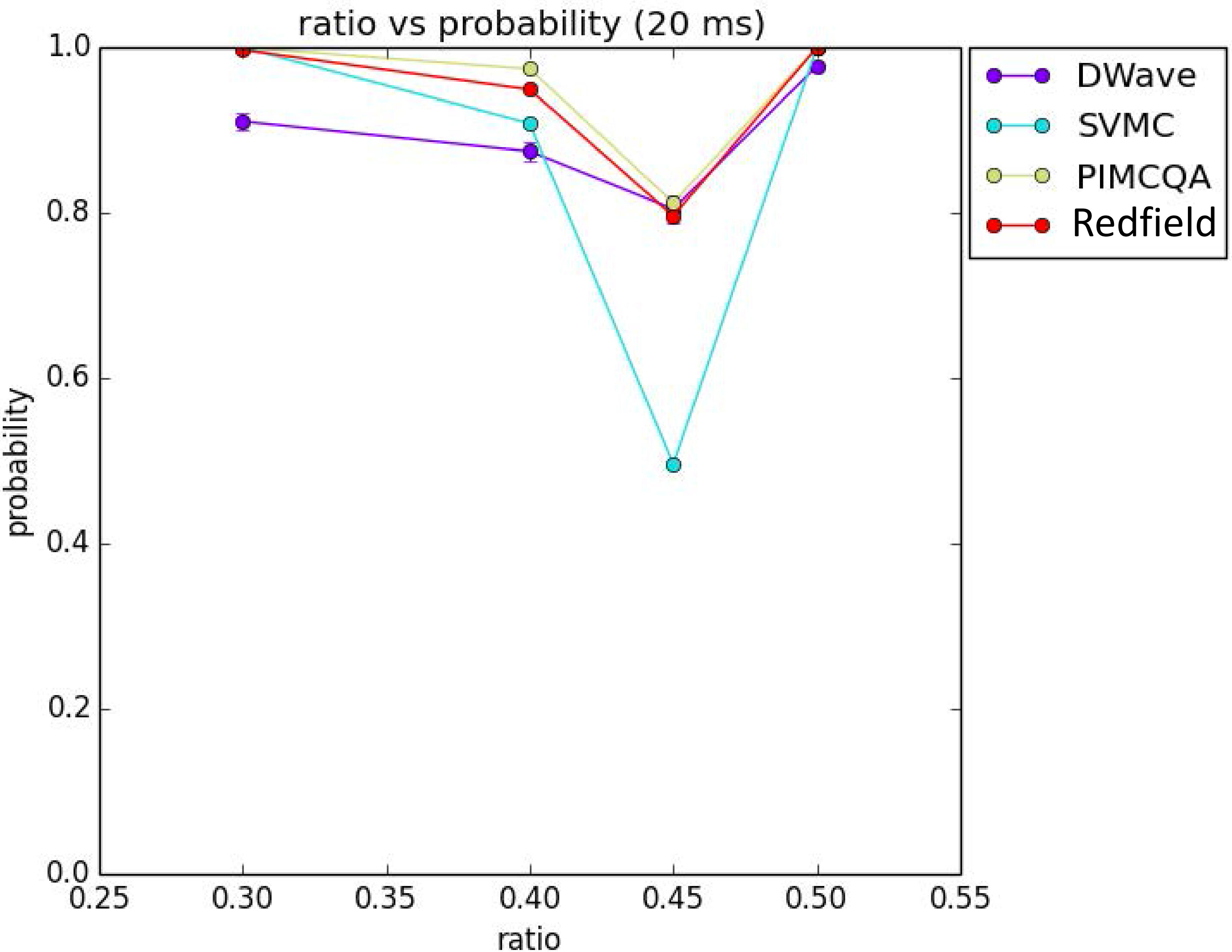
N copies of 2-cluster system

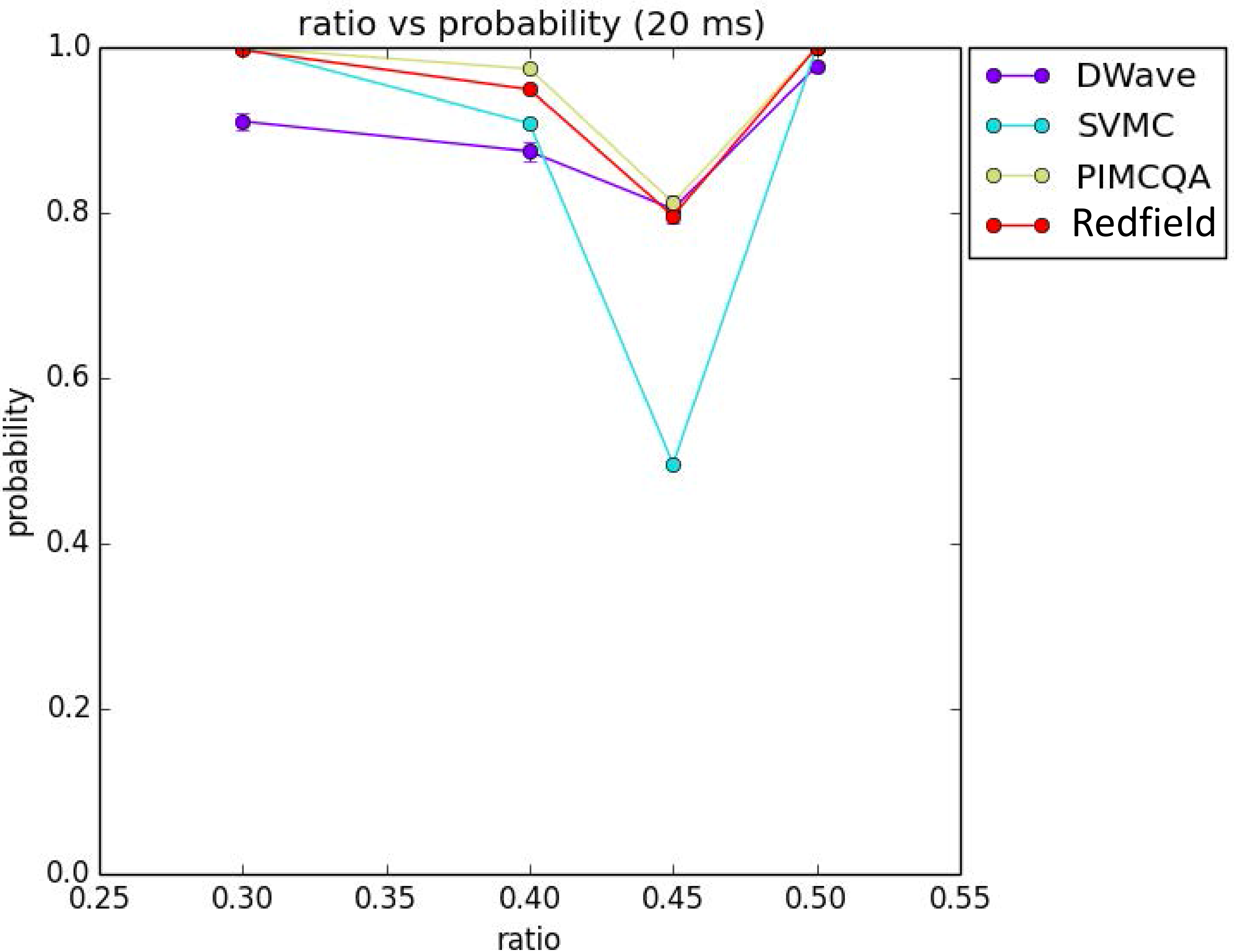


p_0 Probability of success in 2-claster problem

p_0^N Probability of success in N copies of the problem

$$p_0 = 1 - \varepsilon$$
$$p_0^N = e^{-\varepsilon N} \Rightarrow \varepsilon = O(1/N)$$


ratio vs probability (20 ms)
DWave
SVMC
PIMCQA
Redfield
probability
ratio
1.0
0.8
0.6
0.4
0.2
0.0
0.25
0.30
0.35
0.40
0.45
0.50
0.55


ratio vs probability (20 ms)
DWave
SVMC
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Redfield
probability
ratio
1.0
0.8
0.6
0.4
0.2
0.0
0.25
0.30
0.35
0.40
0.45
0.50
0.55

Fully-Connected Graphs: The Sherrington-Kirkpatrick Model

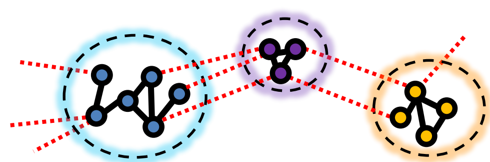
$$H_{SKM} = \sum_{\alpha,\beta} J_{\alpha\beta} S^Z_\alpha S^Z_\beta$$

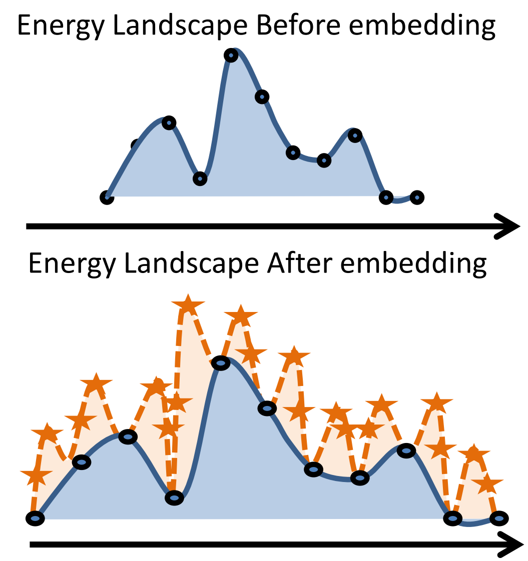
Results to appear in:

“Quantum Optimization of fully-connected Spin glasses”

D. Venturelli, S. Mandra, S. Knysh, B. O’Gorman, V. Smelyankiy

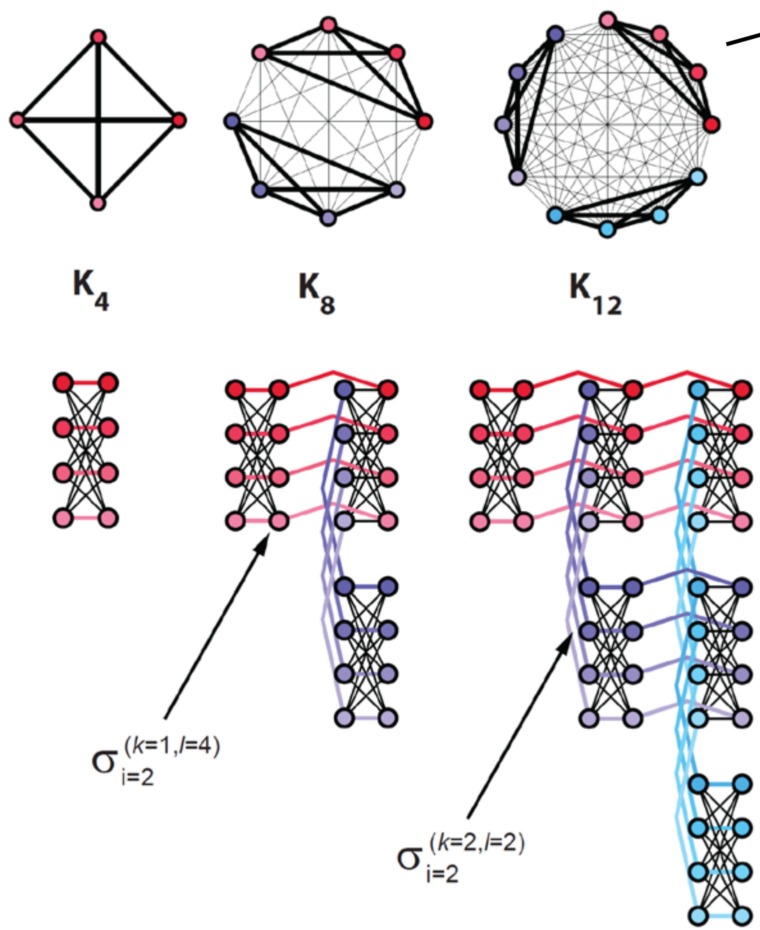
Parameter Setting





Embedding of a Fully-Connected Graph

Systematic Rule Embedding (V. Choi)



- Logical: 4n
- Hardware 4n(n+1)

$$\left(B(t)\sum_{\alpha} S^X_\alpha + A(t)\sum_{\alpha,\beta} J_{\alpha\beta} S^Z_\alpha S^Z_\beta \right)$$

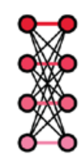
$$\boxed{B(t)\sum_{kli}\sigma_i^{x(kl)}}$$

$A(t)$

Ferromagnetic couplings =1

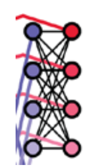
$$\boxed{\sum_{kli}\Big[-\sigma_i^{z(kl)}\sigma_{i+1}^{z(kl)}}$$

Inter-color couplings



$$\boxed{-\sum_{l'<l}\frac{J_{(kl,kl')}}{J_F}\left(\sigma_k^{z(kl)}\sigma_{k+1}^{z(kl')}\right)}$$

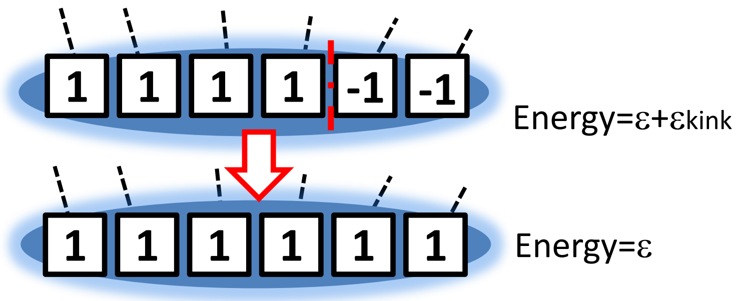
Intra-color couplings

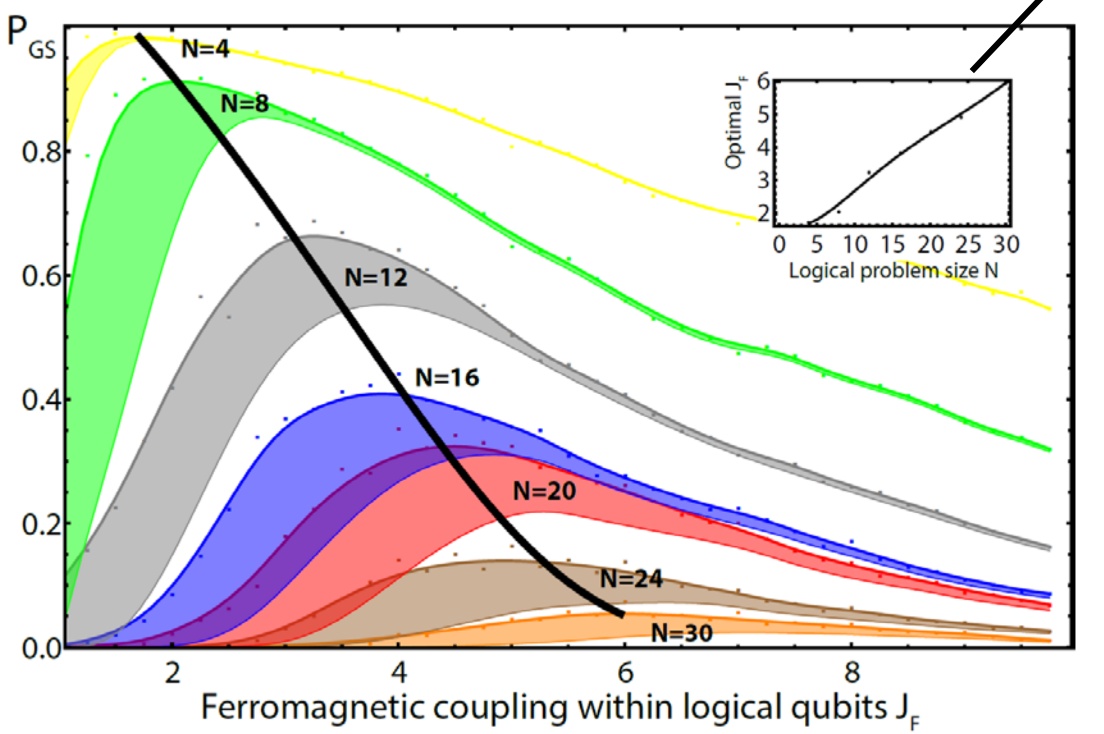


$$\boxed{-\sum_{k'=k+i}\frac{J_{(kl,k'l)}}{J_F}\left(\sigma_i^{z(kl)}\sigma_k^{z(k'l)}\right)\Big]}$$

Optimal Parameter Setting: J_F scales as $N^{1/2}$

Error-Correction





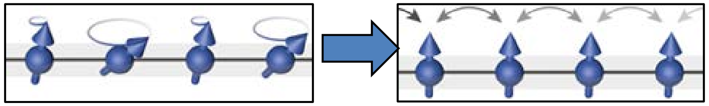
Optimal scaling scales with N :

Ferromagnetic criticality crossover of the chains

VS finite-size criticality of the SKM

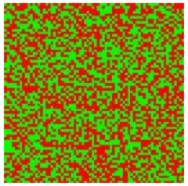
Ferromagnetic transition intrachain

$B(\tau) = A(t)$



Spin-Glass Transition SKM

$B(\tau) = 1.5\, N^{1/2} A(t) / J_F$

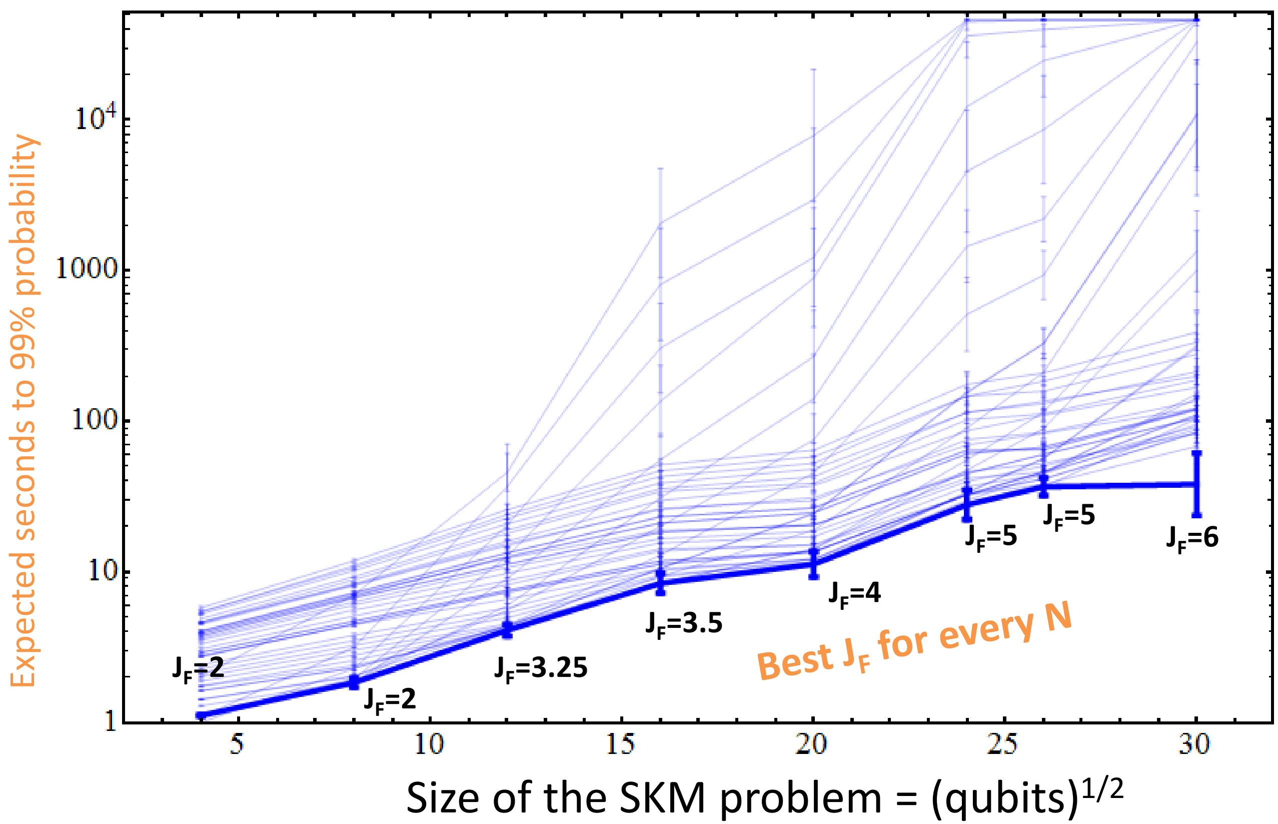


Scaling of the expected runtime

Probability to find the ground state after 1 annealing run (20μs): P_{GS}

Probability to find the ground state after R repetitions: $P^X = 1-(1-P_{GS})^R$

Expected number of repetitions to solve with 99% prob: $R^{99} = \log(0.01)/\log(1-P_{GS})$



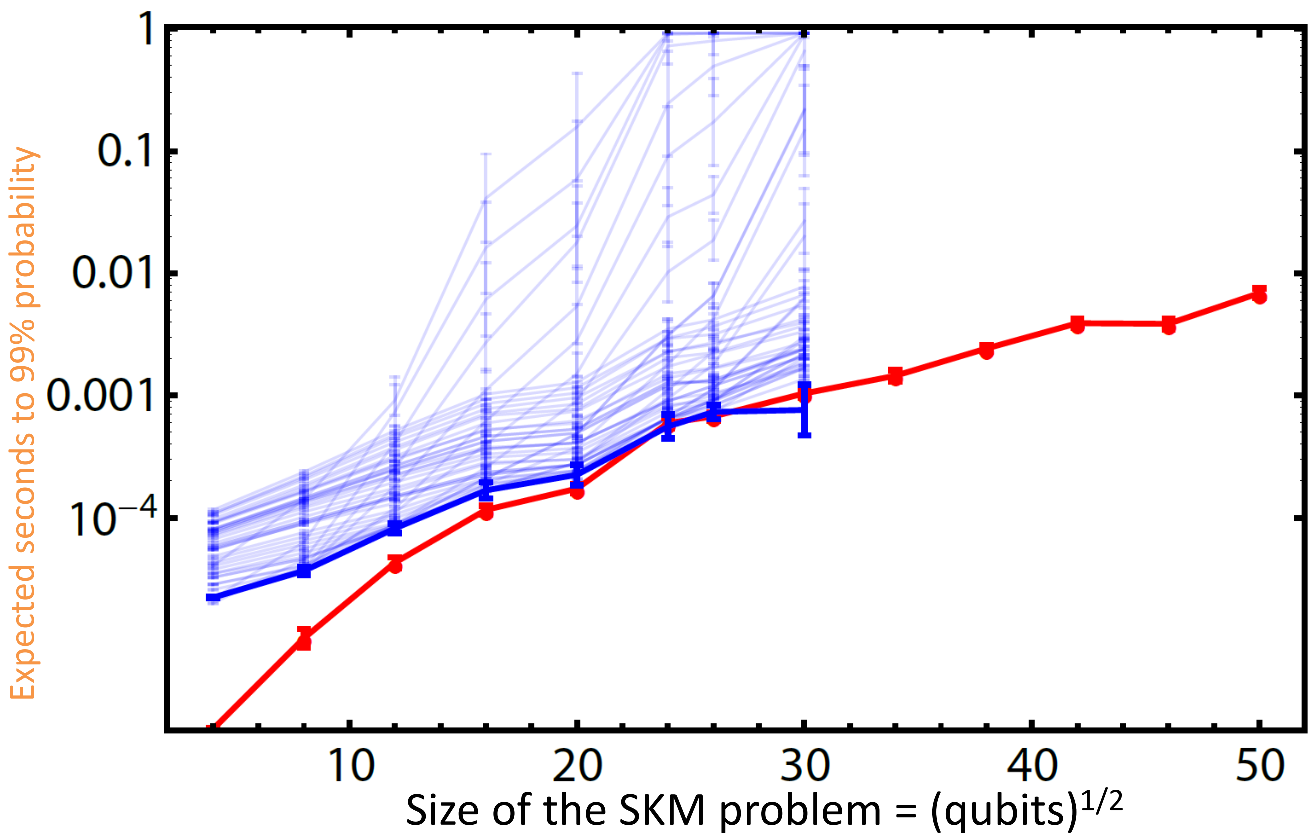
- Tested on 80 random instances, enough to sample difficulty distribution for **median**.
- Ground state checked with exact enumeration method.
- Computed median runtime to get 99% probability of success.
- Errorbars obtained with resampling method to indicate uncertainties.
- Averaging over 12 gauges, each instance run 500,000 times.

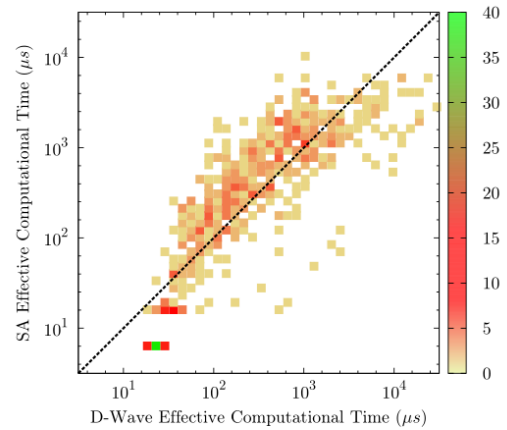
Simulated annealing comparison /correlation

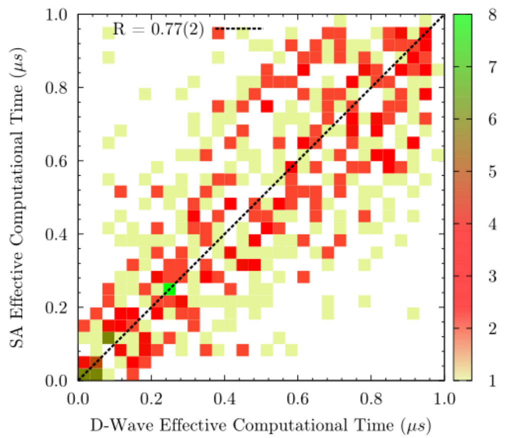
- Simulated Annealing Algorithm: accept solution with Boltzmann Probability

$$P(\Delta E) = \exp\left(-\frac{\Delta E}{k_B T}\right)$$

- Correlation and copula is rather high: **we can use SA to estimate impact of noise**







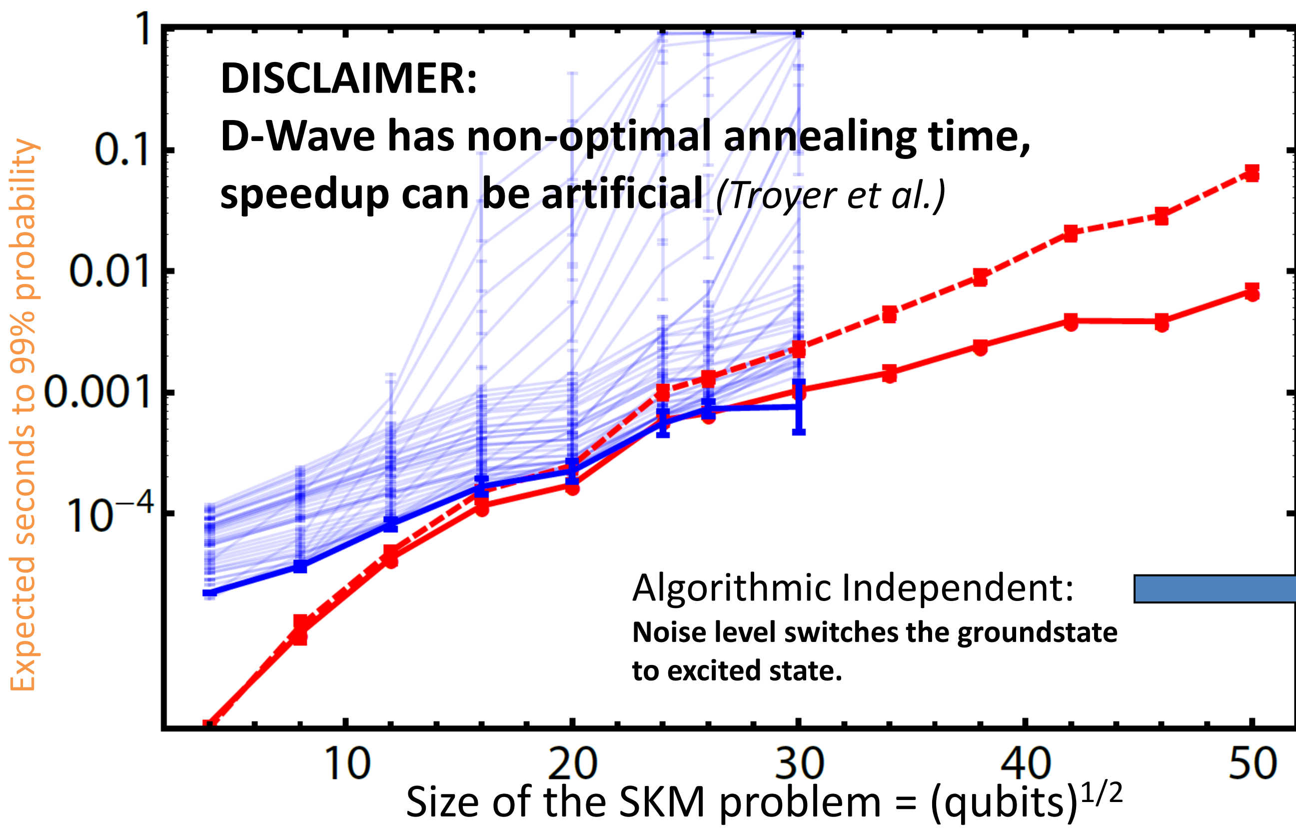
Spoiled performance due to (static) noise

WE ADD NOISE TO SIMULATED ANNEALING: masked speedup?

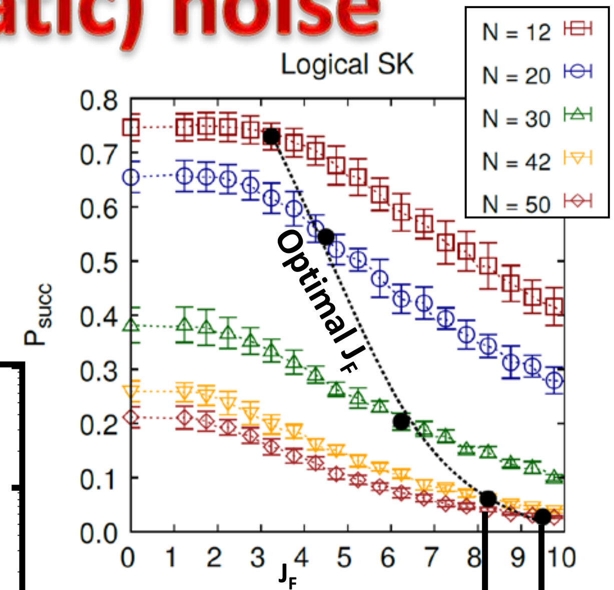
$$H_{dev} = H_{ideal} + J_F \left[\sum_{\alpha\beta} \xi_J^{\alpha\beta} S_\alpha S_\beta + \sum_{\alpha} \xi_h^{\alpha\beta} S_\alpha \right]$$

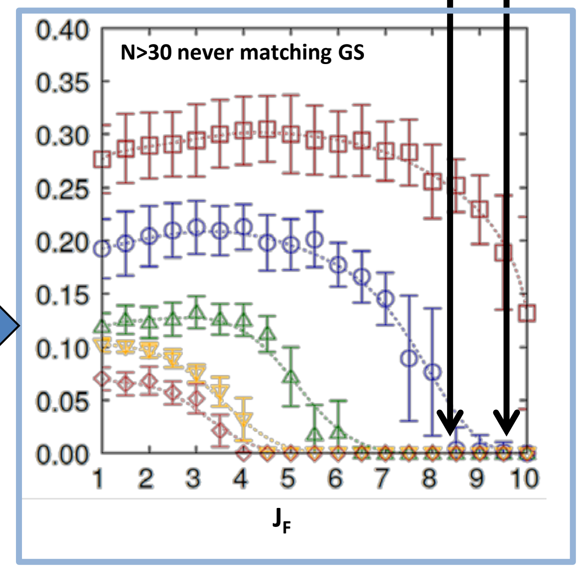
= 0.035 = 0.05

DISCLAIMER:
D-Wave has non-optimal annealing time, speedup can be artificial *(Troyer et al.)*



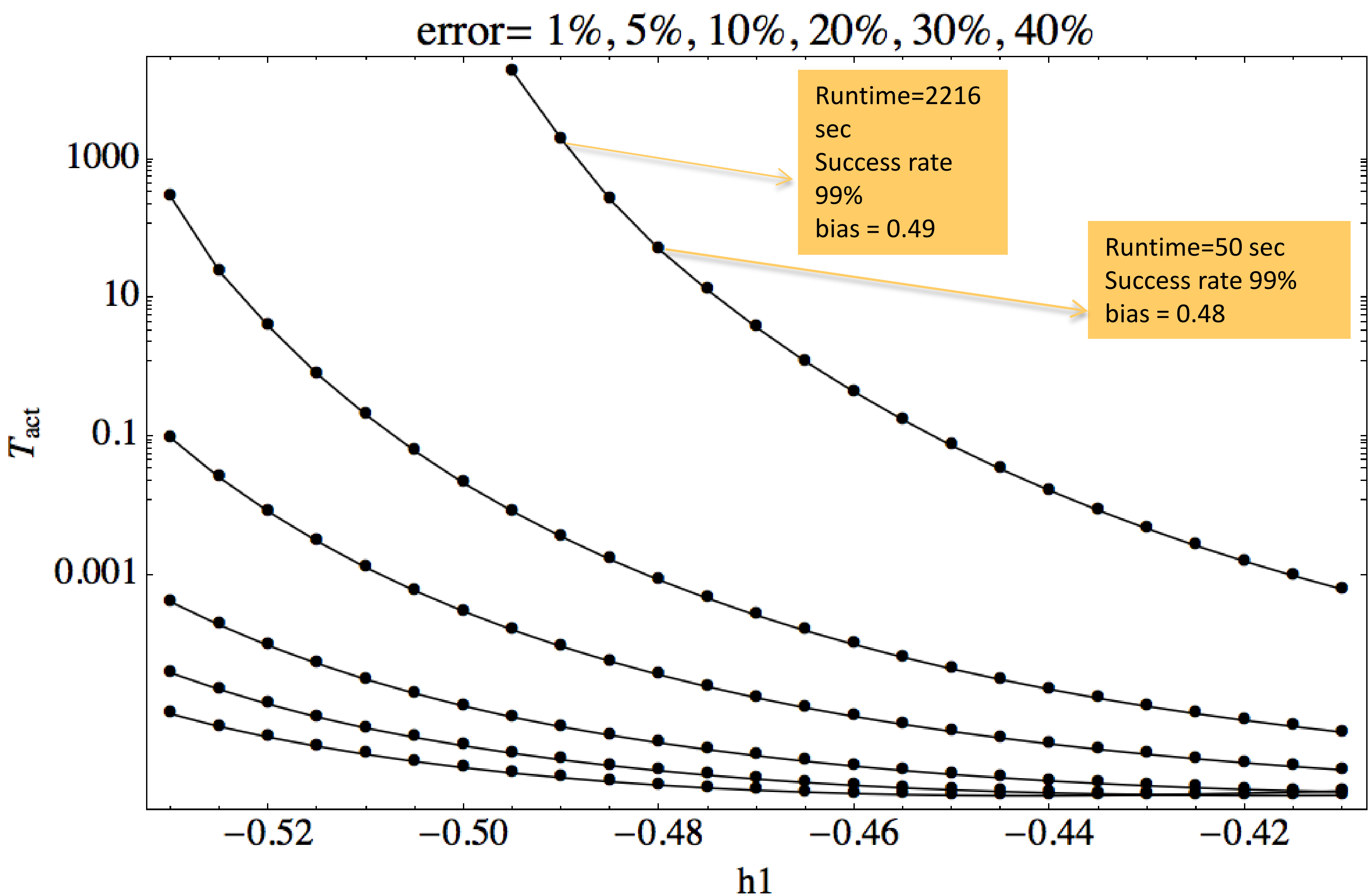
Algorithmic Independent:
Noise level switches the groundstate to excited state.





Conclusions

- Mapped SKM into D-Wave: first benchmark on structured Spin-Glass problems
- Optimized Embedding parameter settings: scaling of J_F connected to criticality
- Compared with Model of Noise in SA, scaling is masked


error= 1%, 5%, 10%, 20%, 30%, 40%
Runtime=2216 sec
Success rate 99%
bias = 0.49
Runtime=50 sec
Success rate 99%
bias = 0.48
1000
10
0.1
0.001
T_{act}
−0.52
−0.50
−0.48
−0.46
−0.44
−0.42
h1