Error Corrected Quantum Annealing with Hundreds of Qubits

D-Wave 2, 512-qubit “Vesuvius” processor
Acquired by Lockheed Martin Corp.
Installed at USC’s Information Sciences Institute (ISI)
Operational since March 2013
D-Wave Two Processor Graph
503/512 functional qubits with “Chimera graph” couplings

D-Wave Two (Vesuvius): 8x8 unit cells comprising 512 qubits

flux qubit
Equipment at USC

The D-Wave One (Rainier) and D-Wave Two (Vesuvius) processors implement AQO with superconducting hardware.

D-Wave One
2012
108 qubits

D-Wave Two
2013
503 qubits

Quantum Annealing for Optimization

Solve a hard problem by slowly changing the system Hamiltonian.

Limitations on the problem type make this an optimization rather than a generalized adiabatic quantum computation.

\[ H(t) = A(t)H_X + B(t)H_{\text{Ising}} \]

\[ H_X = \sum_i X_i \]

\[ H_{\text{Ising}} = \sum_i h_i Z_i + \sum_{i,j} J_{ij} Z_i Z_j \]
Motivation

Hard problems are characterized by small gaps.

Quantum annealing problems with small gaps are more susceptible to errors:

- non-adiabatic transitions
- thermal excitations
- problem mis-specification

As hardware grows, error correction becomes a necessity.

Existing error correction schemes cannot be implemented within our limitations.

We need a new approach.

Quantum Annealing Correction

Goal: an *implementable* encoding to protect an AQO using just 1- or 2-local terms

Key Insight: we can implement the classical 3-bit repetition code, hence can correct against bit-flip errors

\[
H(t) = A(t) \sum_i X_i + B(t) H_{\text{Ising}}
\]

Unencoded:

\[
H_{\text{Ising}} = \sum_i h_i Z_i + \sum_{i,j} J_{ij} Z_i Z_j
\]

Encoded:

\[
\overline{H}_{\text{Ising}} = \alpha \left( \sum_i h_i Z_i + \sum_{i,j} J_{ij} Z_i Z_j \right) + \beta H_{\text{penalty}}
\]

\[
\overline{Z}_i = Z_{i,1} + Z_{i,2} + Z_{i,3}
\]

\[
\overline{Z}_i Z_j = Z_{i,1} Z_{j,1} + Z_{i,2} Z_{j,2} + Z_{i,3} Z_{j,3}
\]

Penalty:

\[
H_{\text{penalty}} = -\sum \text{stabilizer generators}
\]

\(\alpha = \text{problem scale}\)
\(\beta = \text{penalty scale}\)

*factor of 3 gain in energy scale by using 3 biases/couplers*
Construction of $H_{\text{penalty}}$

For implementation reasons, we add a penalty qubit to each logical group and construct $H_{\text{penalty}}$ from the stabilizer generators for the 4-qubit repetition code.

$$H_{\text{penalty}} = \sum_i - \left( Z_{i,1}Z_{i,4} + Z_{i,2}Z_{i,4} + Z_{i,3}Z_{i,4} \right)$$
Construction of $H_{\text{penalty}}$

A bit flip on a single qubit constitutes a decodable error.

$$H_{\text{penalty}} = \sum_i -\left(Z_{i,1}Z_{i,4} + Z_{i,2}Z_{i,4} + Z_{i,3}Z_{i,4}\right)$$

penalty: $2B(t)\beta$  
penalty: $6B(t)\beta$
Construction of $H_{\text{penalty}}$

If two problem qubits are flipped, the error is undecodable

$$H_{\text{penalty}} = \sum_i -\left( Z_{i,1}Z_{i,4} + Z_{i,2}Z_{i,4} + Z_{i,3}Z_{i,4} \right)$$

penalty: $4B(t)\beta$  
penalty: $4B(t)\beta$
Yield was much higher for this processor, resulting in much more regularity and flexibility in the encoded graph.
Antiferromagnetic Chain Cases

Unprotected

\[ H_{\text{Ising}}(\alpha) = \alpha \sum_{i=1}^{N-1} Z_i Z_{i+1} \]

Classical repetition

\[ \overline{H}_{\text{Ising}}(\alpha) = \alpha \sum_{j=1}^{4} \sum_{i=1}^{N-1} Z_{i,j} Z_{i,j+1} \]

Energy penalty only

\[ \overline{H}_{\text{Ising}}(\alpha, \beta) = \alpha \sum_{i=1}^{N-1} Z_i Z_{i+1} + \beta H_{\text{penalty}} \]

Quantum Annealing Correction

Repetition, energy penalty, and majority vote decoding combined
Vesuvius Experimental Results
“hot” problem scale $\alpha=0.3$

![Graph showing the probability of correct answer against antiferromagnetic chain length $N$ or $\bar{N}$, with data points and a fitted line.](image)
Vesuvius Experimental Results

“hot” problem scale $\alpha=0.3$
Vesuvius Experimental Results

problem scale $\alpha=0.3$

Antiferromagnetic chain length $N$ or $\bar{N}$

- Unprotected (U)
- No penalty (NP)
- Encoded with penalty (EP)

(ground state only)
Vesuvius Experimental Results
“hot” problem scale $\alpha=0.3$

run 4 parallel chains, take the best

agrees with $1 - (1 - p_U)^4$
Vesuvius Experimental Results
“hot” problem scale $\alpha=0.3$

![Graph showing the probability of correct answer against antiferromagnetic chain length $N$ or $\bar{N}$.](image)

- Unprotected (U)
- Classical (C)
- No penalty (NP)
- Encoded with penalty (EP)
- Complete (QAC)

(with decoding)
Vesuvius Experimental Results
“cool” problem scale $\alpha=1$

Probability of correct answer

Antiferromagnetic chain length $N$ or $\overline{N}$

Unprotected (U)

fit
Vesuvius Experimental Results
“cool” problem scale $\alpha=1$

![Graph showing probability of correct answer vs. antiferromagnetic chain length N or $\bar{N}$]
Vesuvius Experimental Results
“cool” problem scale $\alpha=1$

![Graph showing the probability of correct answer against antiferromagnetic chain length $N$ or $\bar{N}$.
- Unprotected (U)
- No penalty (NP)
- Encoded with penalty (EP)
- Fit]

Probability of correct answer vs. Antiferromagnetic chain length $N$ or $\bar{N}$.
Vesuvius Experimental Results
“cool” problem scale $\alpha=1$
Vesuvius Experimental Results
“cool” problem scale $\alpha=1$

![Graph showing the probability of correct answer versus antiferromagnetic chain length $N$ or $\bar{N}$ for different classes: Unprotected (U), Classical (C), No penalty (NP), Encoded with penalty (EP), Complete (QAC).]
Energy Penalty Tradeoff

The minimum gap widens and occurs earlier with increasing penalty scale $\beta$.

However, if $\beta$ is large enough to make the first excited state undecodable, the benefit of the energy penalty is lost.
Benchmarking QAC

What about problems that don’t suffer from the domain wall issue? We expect to see better performance of QAC here.

Generate 1000 random problem instances over an encoded graph. Check classical and quantum encoding performance.

Set each coupling from \{-1, -5/6, ..., +5/6, +1\}

Much denser connectivity than chains.

Cannot find alternate embeddings; must use gauges instead.

Scaling Results from Benchmark

QAC and classical repetition separate for $N=112$. The advantage gained from QAC is larger for harder problems.
Robustness of QAC

Previous chain results on the Rainier processor showed surprising performance given that three out of 16 penalty qubits were missing.

If we remove an increasing percentage of penalty qubits from the encoded problems in the benchmark set, we can investigate the extent of the resilience effect.

Create “bridge” qubits to increase code utility
Robustness Scaling Results

- 50 percentile
- 75 percentile
- 90 percentile
- 95 percentile

- Classical QAC
- 30% missing
- 60% missing
- 90% missing

Graphs show the time to solution (number of annealing cycles) for different percentages of missing data, with classical and QAC methods compared.
Role of Penalty Strength in Robustness

As the number of missing penalty qubits grows, the optimal penalty strength for the remaining qubits also increases.

Larger beta compensates for missing penalty qubits, up to a point.
Conclusions

We have developed an implementable error correction scheme for the D-Wave quantum annealing devices and tested it experimentally.

We see excellent performance for antiferromagnetic chains and promising results for large benchmark problems compared to classical repetition using the same resources.

The quantum stabilizer terms are crucial to the error correction/suppression effort because they alter the spectrum during the calculation.

The scheme is also robust to qubit loss, if the missing physical qubits can be embedded as penalty qubits within logical groups.
Thank You