Unitary dynamics in strongly Non-Unitary systems

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QIP relies on the ability to generate, preserve & manipulate Q-coherence & Q-entanglement

"Traditional View"

Dissipative dynamics: quantum coherence/entanglement (typically) Destroyed **= Worst enemy** of **QIP**

Fight with: quantum error correction, decoherence-free Subspaces, dynamical-decoupling, topological-geometric **QIP,**...

"Novel View"

Dissipative dynamics can be *harnessed & exploited* to the end of **QIP:** pure-state preparation, quantum computation, quantum simulations

After all we can make an ally of any enemy, perhaps.....

Setting the Stage

$\mathbf{L}=$ Liouvillian super-operator

$\{\mathbf{E}_t\}_{t\geq 0}=$ Evolution semi-group (positive maps)

$$\frac{\partial \mathbf{E}_t}{\partial t}=\mathbf{L}\mathbf{E}_t:=(\mathbf{L}_0+T^{-1}\tilde{\mathbf{K}})\mathbf{E}_t$$

$\mathbf{L}_0=$ Dissipative generator

$\tilde{\mathbf{K}}(\rho):=-i[\tilde{K},\rho],\qquad \|\tilde{K}\|=O(1)$

Basic Equation: dissipation + small coherent term (T adiabatic time-scale)

Steady-State Manifold (***SSM***): $\rho:\quad \mathbf{L}_0(\rho)=0$

Q: Can we coherently manipulate states inside the *SSM*?

The Projection Theorem

[PZ & L Campos Venuti, arXiv:1404.4673]

$\mathbf{P}_0 =$ Spectral projection onto $\ker(\mathbf{L}_0)$ $(\mathbf{P}_0\mathbf{L}_0 = \mathbf{L}_0\mathbf{P}_0 = 0)$ $\tilde{\mathbf{K}}_{eff} = \mathbf{P}_0\tilde{\mathbf{K}}\mathbf{P}_0$

$$\|(\mathbf{E}_T - e^{\tilde{\mathbf{K}}_{eff}})\mathbf{P}_0\| = O(1/T) \Rightarrow \lim_{T\to\infty} \mathbf{E}_T\mathbf{P}_0 = e^{\tilde{\mathbf{K}}_{eff}}\mathbf{P}_0$$

$\|(\mathbf{1}-\mathbf{P}_0)\mathbf{E}_T\mathbf{P}_0\| = O(1/T)$ Transitions outside the ***SSM*** are suppressed

Non SS are adiabatically decoupled and the dynamics inside the ***SSM*** is ruled by the effective projected generator $\tilde{\mathbf{K}}_{eff}$

- Fermi-Golden rule at infinite order in the perturbation K

Who's $\tilde{\mathbf{K}}_{eff}$?

Projection Theorem: a simple argument

1st order Dyson's series

$$\mathbf{E}_t\mathbf{P}_0 = \mathbf{P}_0 + \frac{t}{T}\ \mathbf{P}_0\tilde{\mathbf{K}}\mathbf{P}_0 \ + \ \frac{1}{T}\int_0^t e^{t\mathbf{L}_0}\mathbf{Q}_0\tilde{\mathbf{K}}\mathbf{P}_0 dt \ + \ O(T^{-2})$$

$$\int_0^T e^{t\mathbf{L}_0}\mathbf{Q}_0\tilde{\mathbf{K}}\mathbf{P}_0 dt = \mathbf{S}(e^{T\mathbf{L}_0} - \mathbf{1})\tilde{\mathbf{K}}\mathbf{P}_0 \qquad \mathbf{S} := -\int_0^\infty e^{t\mathbf{L}_0}\mathbf{Q}_0 = \text{"}\mathbf{L}_0^{-1}\text{"}$$

$$\mathrm{Re}(\lambda_h) < 0 \quad (h > 0)$$

$$\| \mathbf{E}_T\mathbf{P}_0 - e^{\mathbf{P}_0\tilde{\mathbf{K}}\mathbf{P}_0}\mathbf{P}_0 \| \le 2\frac{\|\mathbf{S}\|}{T}\|\tilde{\mathbf{K}}\| + O(T^{-2}), \qquad \|\mathbf{S}\| = O(\Delta^{-m_h})$$

- Deviations from adiabaticity i.e., transitions outside the **SSM** controlled by dissipative gap $\Delta := \min_{h>0} |\mathrm{Re}\,\lambda_h|$

- Dynamical phase transitions, info. geometry of diss. **QPT**s

L. Banchi, P. Giorda & PZ, Phys. Rev. E ***89****, 022102 (2014)*

Example 1

$$\mathbf{L}_0(\rho) = \Phi(\rho) - \rho, \qquad \Phi(\rho) = \sum A_i \rho A_i^+, \ \ \Phi(\mathbf{1}) = \mathbf{1} \quad \Phi^*(\mathbf{1}) = \mathbf{1}$$

$$A := A\lg[A_i, A_i^+] \ \Rightarrow \ \ker \mathbf{L}_0 = A^{'} \cong \oplus_J M(n_J) \otimes \mathbf{1}_{d_J}$$

Dissipator kernel = commutant algebra (Kribs 2006)

SSM: convex hull of *A*-noiseless states (mixed in general)

$$\mathbf{P}_0(X) = \int_{U \in \mathbf{U}(A)} UXU^+ dU = \sum_J Tr_{d_J}(\Pi_J X \Pi_J) \otimes \mathbf{1}_{d_J} \in A^{'}$$

= *A*-symmetrization

$$\tilde{\mathbf{K}}_{eff}(X) = -i[\tilde{K}_{eff}, X], \ \ \tilde{K}_{eff} = P_0(\tilde{K}) \quad (X \in A^{'})$$

Effective generator: environment-symmetrized *K*

Example 2: one free system coupled to a dissipative one

$$\mathbf{L}_0 = \mathbf{1}_A \otimes \mathbf{L}_B, \qquad \exists! \rho_B \, / \;\; \mathbf{L}_B(\rho_B) = 0 \;\; \Rightarrow \qquad \mathbf{L}_0(X \otimes \rho_B) = 0 \quad \forall X$$

$$\tilde{\mathbf{K}}_{eff}(X) = -i[\tilde{K}_{eff}, X], \qquad \tilde{K}_{eff} = Tr_B[(\mathbf{1}_A \otimes \rho_B)K] \otimes \mathbf{1}_B$$

Example 3: **SSM** $\supset$ a Decoherence-Free Subspace

$$|\phi\rangle, |\phi'\rangle \in C \subset H, \;\; |\varphi\rangle \in C^{perp} \quad \Rightarrow \mathbf{L}_0(|\phi\rangle\langle\phi'|) = 0 \qquad \mathbf{P}_0(|\phi\rangle\langle\varphi|) = \mathbf{P}_0(|\varphi\rangle\langle\phi|) = 0$$

$$X \in L(C) \;\; \Rightarrow \qquad \tilde{\mathbf{K}}_{eff}(X) = -i[\tilde{K}_{eff}, X] \qquad \tilde{K}_{eff} = \Pi_C \tilde{K} \Pi_C$$

Effective dynamics over the **DFS** ruled by a projected Hamiltonian

Remarks I

- For Abelian *A:* $\mathbf{P}_0(X) = \sum_J \Pi_J X \Pi_J$ = meas. super-op: ***Zeno** dynamics*

  *G. A. Paz-Silva et PRL **108**, 080501 (2012); D. Burgath et al, arXiv:1403.5752*

- Rotating frame generated by ***K:*** no Hamiltonian term & (slowly) time-dependent bath: The Projection Theorem is an adiabatic th. for open Sys (i.e. system's always inside the instantaneous SSM)

  *M. S. Sarandy , D. A. Lidar, PRA **73**, 062101 (2006)*

- Effective evolution in the SSM is geometric ➔ dissipation-assisted Holonomic Quantum Computation

  *A. Carollo et PRL **96**, 020403 (2006); O. Oreshkov, J. Calsamiglia, PRL, **105** 050503 (2010)*

Remarks II

- Effective dynamics may be *more complex* that the bare one:
  More-non local, higher-computational power e.g.,

$$[K_1, K_2] = 0 \quad but \quad [\mathbf{P}_0(K_1), \mathbf{P}_0(K_2)] \neq 0$$

D. Burgath et al, arXiv:1403.5752

- Effective dynamics is robust against perturbations of *L*
  that are removed by the projection e.g.,

$$H - K \in Ker(\mathbf{P}_0) \implies H_{eff} = K_{eff}$$

- If the 1st order effective terms vanishes one has a 2nd order effective (non unitary) dynamics over the **SSM** (up to $O\left(\frac{\|\mathbf{S}\|}{T^{1/2}}\right)$ corrections)

$$\mathbf{P}_0\tilde{\mathbf{K}}\mathbf{P}_0 = 0, \; \|K\| = O(T^{-1/2}) \implies \tilde{\mathbf{K}}_{eff} = -\; \mathbf{P}_0\tilde{\mathbf{K}}\mathbf{S}\tilde{\mathbf{K}}\mathbf{P}_0$$

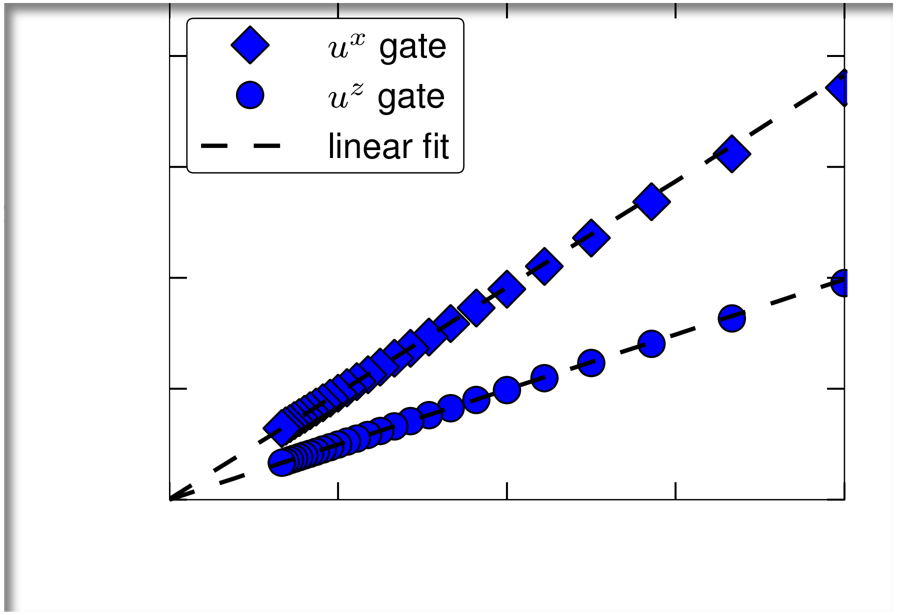
Dissipation-assisted logic gates on a DFS

$$\mathbf{L}_0(\rho) = \sum_{\alpha=x,y,z} \left(S^\alpha \rho S^\alpha - 1/2\{(S^\alpha)^2, \rho\} \right), \qquad S^\alpha := \sum_{j=1}^{N} \sigma_j^\alpha$$

$$N = 4 \quad \Longrightarrow \quad \ker(\mathbf{L}_0) = M(2) \otimes \mathbf{1}_1 \oplus .... \cong L(C) \oplus .... \quad [\dim(\ker \mathbf{L}_0) = 14]$$

$$C := \{ |\phi\rangle \in (\mathbf{C}^2)^{\otimes 4} \, / \, S^\alpha |\phi\rangle = 0, \alpha = x, y, z \}$$

Singlet sector = ***DFS***

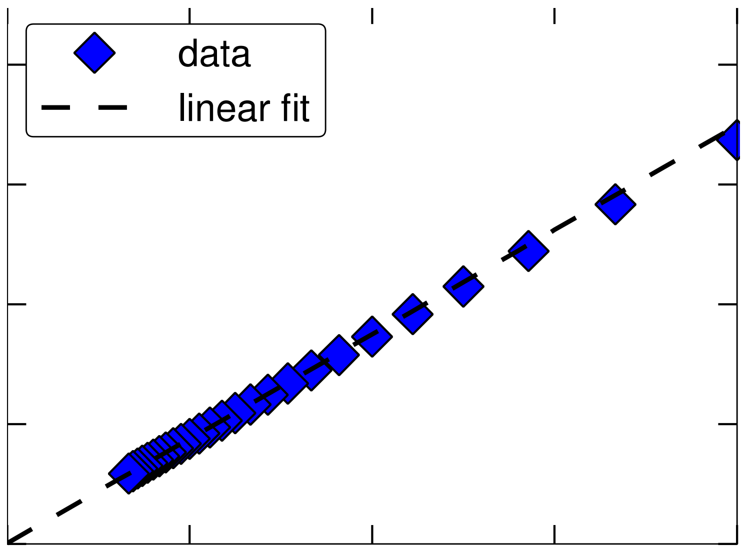


$$\tilde{K}^x = \frac{3}{2}(\sigma_1^z \sigma_2^z + \sigma_2^z \sigma_3^z + 1), \quad \tilde{K}^z = -\frac{\sqrt{3}}{2}(\sigma_1^z \sigma_2^z - \sigma_2^z \sigma_3^z + \sigma_1^z), \quad \Longrightarrow \quad \tilde{K}_{eff}^\alpha \cong \sigma^\alpha$$

Dissipation-assisted logic gates on Noiseless Subsytems

$$\mathbf{L}_0(\rho) = 1/3 \sum_{\alpha=x,y,z} U^{\alpha} \rho (U^{\alpha})^{+} - \rho, \qquad U_{\alpha} := e^{i\vartheta_{\alpha} S^{\alpha}}$$

$N = 3$ $\qquad \ker \mathbf{L}_0 = M(2) \otimes \mathbf{1}_2 \oplus M(1) \otimes \mathbf{1}_4$ $\qquad \dim(\ker \mathbf{L}) = 2^2 + 1^1 = 5$



$$\tilde{K} = \sigma_1^x \sigma_2^x$$

Remark: The ***SSM*** comprises just mixed states

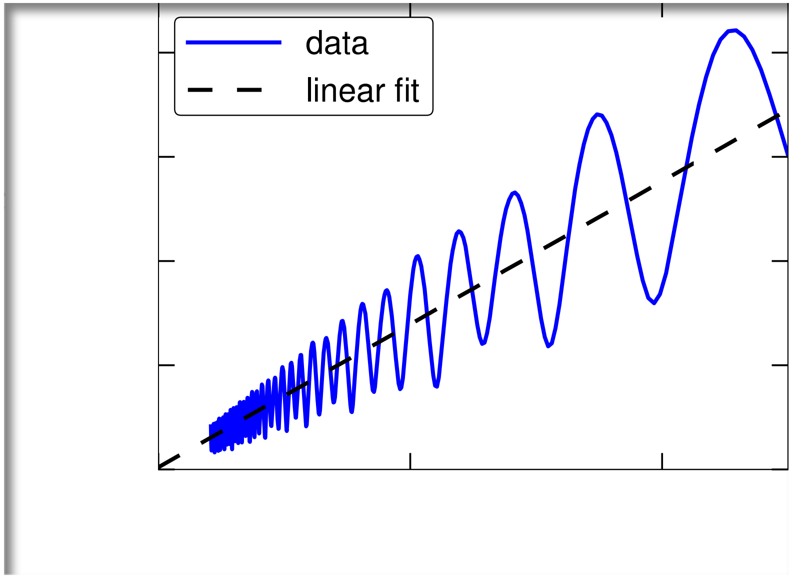
A Unitary example

$$H_0 = S^+ \otimes (\sum_q g_q b_q) + .S^- \otimes (\sum_q g_q b_q^+) + .1 \otimes \sum_q \omega_q b_q^+ b_q$$

$$|\phi_k\rangle := \sum_{j=1}^{N} e^{i\frac{2\pi jk}{N}} \sigma_j^+ |\downarrow,...,\downarrow\rangle \otimes |0\rangle \quad \Rightarrow \quad H_0 |\phi_k\rangle = 0 \quad (k = 1,..., N-1)$$

$$C = span\{|\phi_k\rangle\}_{k=1}^{N-1}$$

Dark state manifold of H_0 *(**DFS** @ T=0)*



$$\tilde{K} = \sigma_1^z \quad \Rightarrow \quad \tilde{K}_{eff} = 2(N-1)|\phi\rangle\langle\phi| \; -1, \qquad |\phi\rangle := \frac{1}{\sqrt{N-1}} \sum_{k=1}^{N-1} e^{-i\frac{2\pi k}{N}} |\phi_k\rangle$$

Projection Th reduces to an adiabatic one for closed sys

Conclusions

- Projection th. over the ***SSM*** of strongly dissipative systems
- Adiabatic coherent (geometric) manipulations on the ***SSM***
- Complexity, (non) locality and robustness can be enhanced
- Dissipation-assisted gates in ***DFSs*** and ***NSs***

Moral: dissipation & decoherence can be good guys...

PZ & L Campos Venuti, arXiv:1404.4673

THANKS FOR THE ATTENTION!

Projection Theorem: Sketch of the proof

P spectral projection of **L** originated by the *0* eigenvalue of **L**$_0$

$$\exp(t\mathbf{L})\mathbf{P} = \exp(t\mathbf{PLP})\mathbf{P} = \exp[t\mathbf{P}_0\mathbf{LP}_0 + t(\mathbf{PLP} - \mathbf{P}_0\mathbf{LP}_0)]\mathbf{P} := e^{\mathbf{X}+\mathbf{Y}}\mathbf{P}$$

$$\mathbf{X} := t\mathbf{P}_0\mathbf{LP}_0 = \frac{t}{T}\mathbf{P}_0\tilde{\mathbf{K}}\mathbf{P}_0$$

$$\mathbf{Y} := t(\mathbf{PLP} - \mathbf{P}_0\mathbf{LP}_0) = -\frac{t}{T^2}\left(\mathbf{P}_0\tilde{\mathbf{K}}\mathbf{P}_0\tilde{\mathbf{K}}\mathbf{S} + \mathbf{P}_0\tilde{\mathbf{K}}\mathbf{S}\tilde{\mathbf{K}}\mathbf{P}_0 + \mathbf{S}\tilde{\mathbf{K}}\mathbf{P}_0\tilde{\mathbf{K}}\mathbf{P}_0\right) + O(\frac{t}{T^3})$$

$$\mathbf{P} = \mathbf{P}_0 + (\mathbf{P} - \mathbf{P}_0) = \mathbf{P}_0 - \frac{1}{T}\left(\mathbf{P}_0\tilde{\mathbf{K}}\mathbf{S} + \mathbf{S}\tilde{\mathbf{K}}\mathbf{P}_0\right) + O(\frac{t}{T^2}) \qquad (Kato)$$

$$\exp(t\mathbf{L})\mathbf{P}_0 = [e^{\mathbf{X}} + (e^{\mathbf{X}+\mathbf{Y}} - e^{\mathbf{X}})]\mathbf{P} - \exp(t\mathbf{L})(\mathbf{P} - \mathbf{P}_0) \;\Rightarrow$$

$$\exp(t\mathbf{L})\mathbf{P}_0 - e^{\mathbf{X}}\mathbf{P}_0 = e^{\mathbf{X}}(\mathbf{P} - \mathbf{P}_0) + (e^{\mathbf{X}+\mathbf{Y}} - e^{\mathbf{Y}})\mathbf{P} - e^{t\mathbf{L}}(\mathbf{P} - \mathbf{P}_0)$$

$$\| \exp(t\mathbf{L})\mathbf{P}_0 - e^{\mathbf{X}}\mathbf{P}_0 \| \le 2\| \mathbf{P} - \mathbf{P}_0 \| + \| e^{\mathbf{X}+\mathbf{Y}} - e^{\mathbf{X}} \| \le 2\| \mathbf{P} - \mathbf{P}_0 \| + \| \mathbf{Y} \| e^{\|\mathbf{X}\|+\|\mathbf{Y}\|}$$

$$\Longrightarrow \qquad \| e^{T\mathbf{L}}\mathbf{P}_0 - e^{\mathbf{P}_0\tilde{\mathbf{K}}\mathbf{P}_0}\mathbf{P}_0 \| = O(\frac{1}{T}) \qquad q.e.d.$$

$$\mathbf{L}_0 = \mathbf{1}_A \otimes \mathbf{L}_B, \qquad \exists!\, \rho_B \,/\; \mathbf{L}_B(\rho_B) = 0 \quad \Longrightarrow \qquad \mathbf{L}_0(X \otimes \rho_B) = 0 \quad \forall X$$

$$\tilde{\mathbf{K}}_{eff}(X) = -i[\tilde{K}_{eff}, X], \qquad \tilde{K}_{eff} = Tr_B[(\mathbf{1}_A \otimes \rho_B)K] \otimes \mathbf{1}_B$$

$$|\phi\rangle \in C \subset H,\; |\varphi\rangle \in C^{perp} \quad \Rightarrow \mathbf{L}_0(|\phi\rangle\langle\phi|) = |\phi\rangle\langle\phi|) \qquad \mathbf{L}_0(|\phi\rangle\langle\varphi|) = \mathbf{L}_0(|\varphi\rangle\langle\phi|) = 0$$

$$X \in L(C) \;\Longrightarrow \qquad \tilde{\mathbf{K}}_{eff}(X) = -i[\tilde{K}_{eff}, X] \qquad \tilde{K}_{eff} = \Pi_C \tilde{K} \Pi_C$$

$$\mathbf{SSM} = Conv - Hull\{\rho_J \otimes \frac{\mathbf{1}_{d_J}}{d_J}\}$$

$$\ker \mathbf{L}_0 \cong \oplus_J M(n_J) \otimes \mathbf{1}_{d_J}, \qquad n_J = \frac{(2J+1)N!}{(N/2+J+1)!(N/2-1)!},$$

$$\mathbf{P}_0(X) = \int_{U \in \mathbf{U}(A)} UXU^+ dU = \sum_J Tr_{d_J}(\Pi_J X \Pi_J) \otimes \mathbf{1}_{d_J} \in A'$$

$$X \in A' \Rightarrow \tilde{\mathbf{K}}_{eff}(X) = -i[\tilde{K}_{eff}, X], \;\; \tilde{K}_{eff} = P_0(\tilde{K})$$

$$\mathbf{E}_t\mathbf{P}_0 = \mathbf{P}_0 + \frac{t}{T}\ \mathbf{P}_0\tilde{\mathbf{K}}\mathbf{P}_0 \quad + \quad \frac{1}{T}\int_0^t e^{t\mathbf{L}_0}\mathbf{Q}_0\tilde{\mathbf{K}}\mathbf{P}_0 dt \quad + \quad O(T^{-2})$$

$$\int_0^T e^{t\mathbf{L}_0}\mathbf{Q}_0\tilde{\mathbf{K}}\mathbf{P}_0 dt = \mathbf{S}(e^{T\mathbf{L}_0} - \mathbf{1})\tilde{\mathbf{K}}\mathbf{P}_0$$

$$\mathbf{S} := -\int_0^\infty e^{t\mathbf{L}_0}\mathbf{Q}_0 = -\sum_{h>0}\left(\frac{\mathbf{P}_h}{(-\lambda_h)} + \sum_{m=1}^{m_h-1}\frac{\mathbf{D}_h^m}{(-\lambda_h)^{m+1}}\right) = "\mathbf{L}_0^{-1}"$$

$$\| \mathbf{E}_T\mathbf{P}_0 - e^{\mathbf{P}_0\tilde{\mathbf{K}}\mathbf{P}_0}\mathbf{P}_0 \| \le 2\frac{\|\mathbf{S}\|}{T}\|\tilde{\mathbf{K}}\| \quad + \quad O(T^{-2})$$

$$\|\mathbf{S}\| = O(\Delta^{-m_h}) \qquad \Delta := \min_{h>0}|\lambda_h|$$

$$\mathbf{R}(z) := (z-\mathbf{L})^{-1} = \sum_h \left(\frac{\mathbf{P}_h}{z-\lambda_h} + \sum_{k=1}^{m_h-1} \frac{\mathbf{D}_h^k}{(z-\lambda_h)^{k+1}} \right) \qquad \mathbf{P}_0\mathbf{L}_0 = \mathbf{L}_0\mathbf{P}_0 = 0$$

$$\mathbf{L} = \sum_h (\lambda_h \mathbf{P}_h + \mathbf{D}_h), \qquad \mathbf{P}_h\mathbf{D}_h = \mathbf{D}_h\mathbf{P}_h = \mathbf{D}_h, \; [\mathbf{P}_h, \mathbf{P}_k] = 0, \; \sum_h \mathbf{P}_h = \mathbf{1}, \; (\mathbf{D}_h)^{m_h} = 0$$

$$\frac{\partial \mathbf{E}_t}{\partial t} = \mathbf{L}\mathbf{E}_t := (\mathbf{L}_0 + T^{-1}\tilde{\mathbf{K}})\mathbf{E}_t \quad \Rightarrow \quad \mathbf{E}_t = e^{t\mathbf{L}}$$

$$\tilde{\mathbf{K}}(\rho) := -i[\tilde{K}, \rho], \qquad \| \tilde{K} \| = O(1)$$

$$\| (\mathbf{E}_T - e^{\tilde{\mathbf{K}}_{eff}})\mathbf{P}_0 \| = O(1/T) \qquad \tilde{\mathbf{K}}_{eff} := \mathbf{P}_0\tilde{\mathbf{K}}\mathbf{P}_0$$

$$\mathbf{E}_t = e^{t\mathbf{L}} = \prod_h e^{t(\lambda_h\mathbf{P}_h + \mathbf{D}_h)} = \mathbf{P}_0 + \sum_{h>0} \left(\mathbf{P}_h e^{t\lambda_h} + e^{t\lambda_h} \sum_{k=1}^{m_h-1} \frac{(t\mathbf{D})^k}{k!} \right)$$