Exact Rate-Distortion in Autoencoders via Echo Noise

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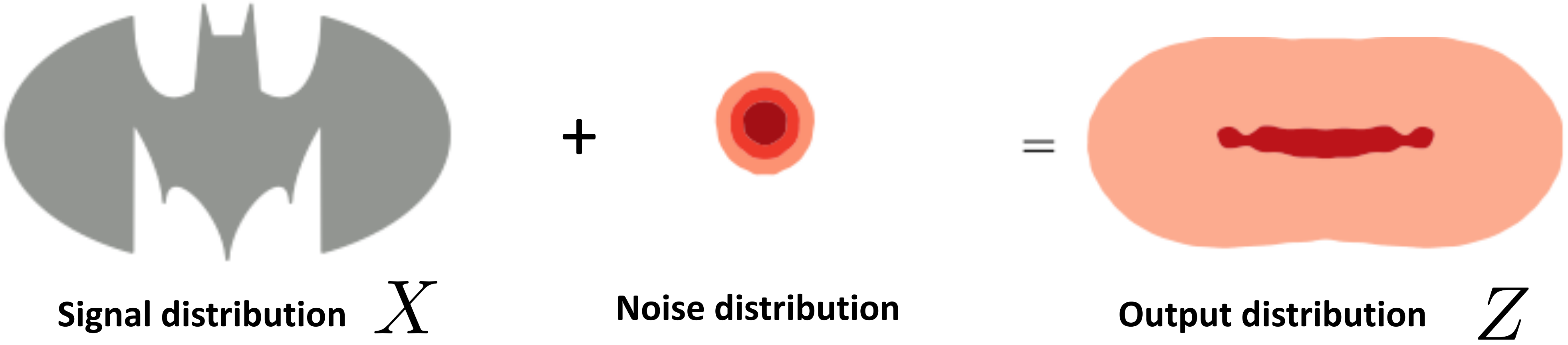
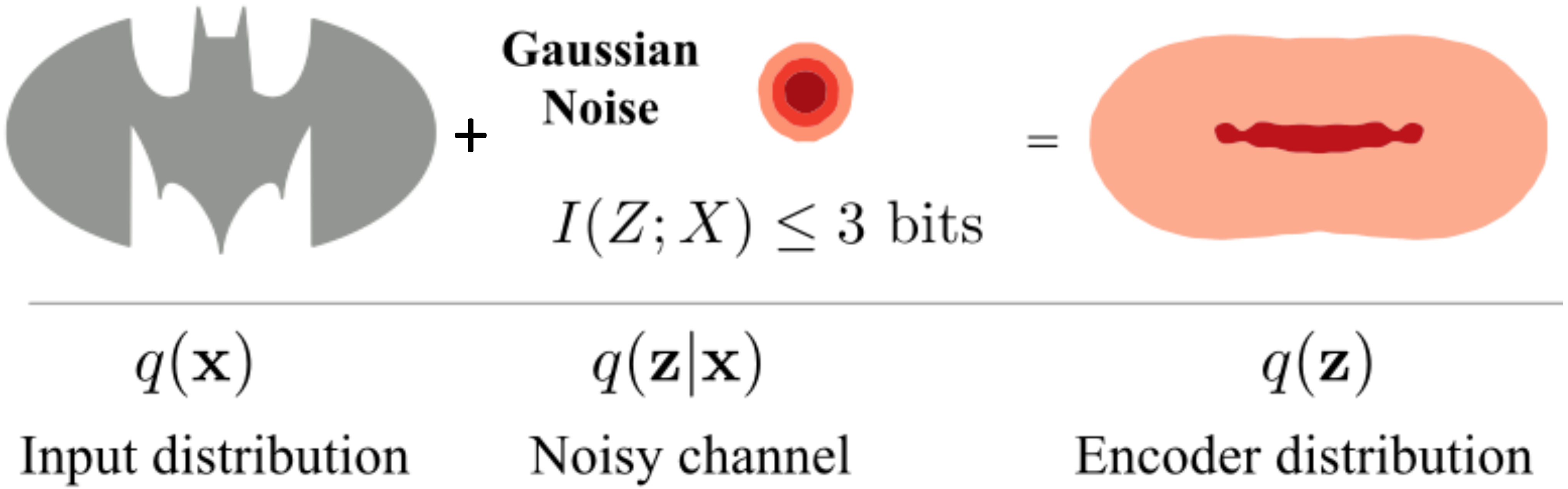
arXiv:1904.07199, NeurIPS 2019



$$I(Z;X) = h(Z) - h(Z|X)$$

Entropy of output

Entropy of noise



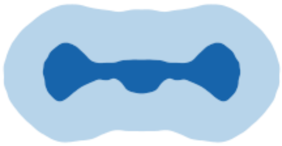
Typical assumptions for mutual information bounds for noisy channel:

- Noise is Gaussian
- Noise is conditionally independent
- Tight only if encoded distribution is Gaussian

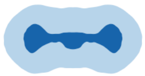
Information in Echo Channel

$$I(Z;X) = h(Z) - h(Z|X)$$

Entropy of output



Entropy of noise



Shape is the same – differ only by a scale function S(x)

$$I(Z;X) = -\mathbb{E} \log \left| \det S(\mathbf{x}) \right|$$

- Exact analytic expression for rate
- S(x) parametrized by any neural network
- No assumptions of Gaussianity or independence in Z

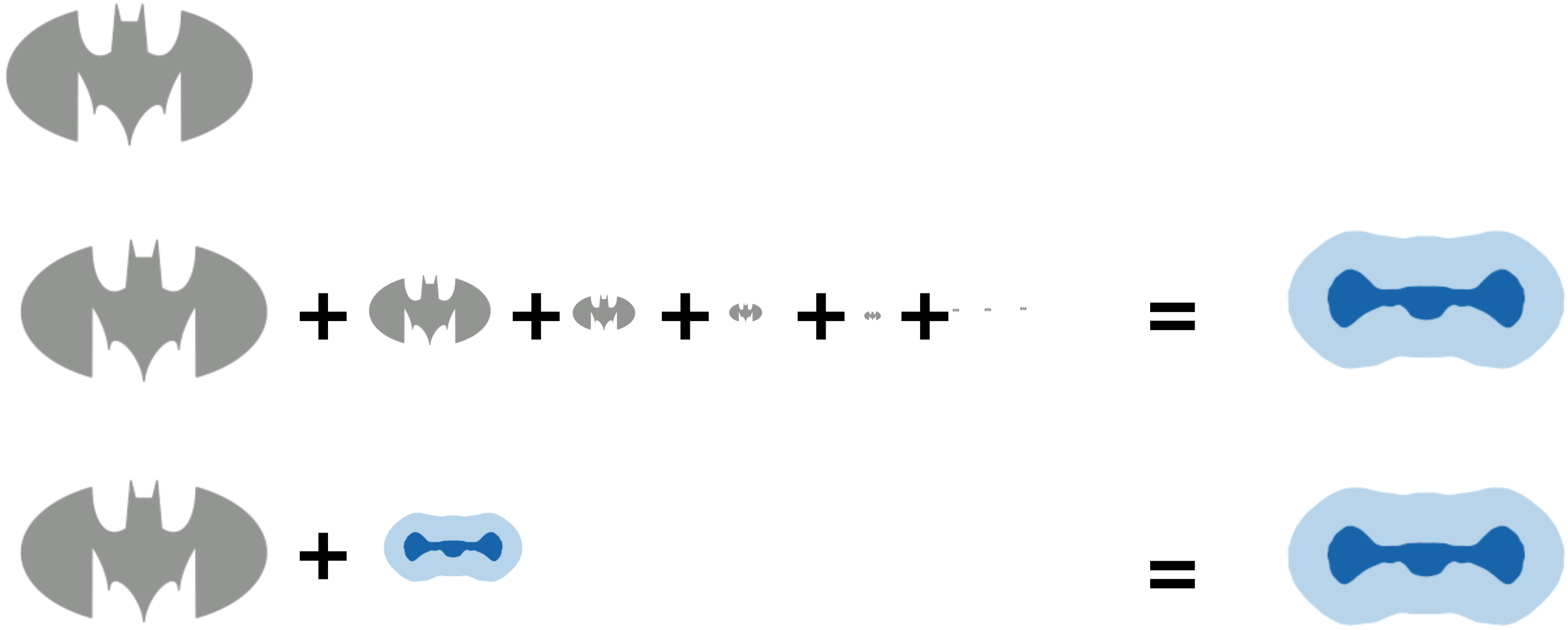
How to sample echo noise to get self similarity

Self-similarity between noise distribution and output distribution is the key

Data drawn iid from this distribution

Sample the echo noise distribution

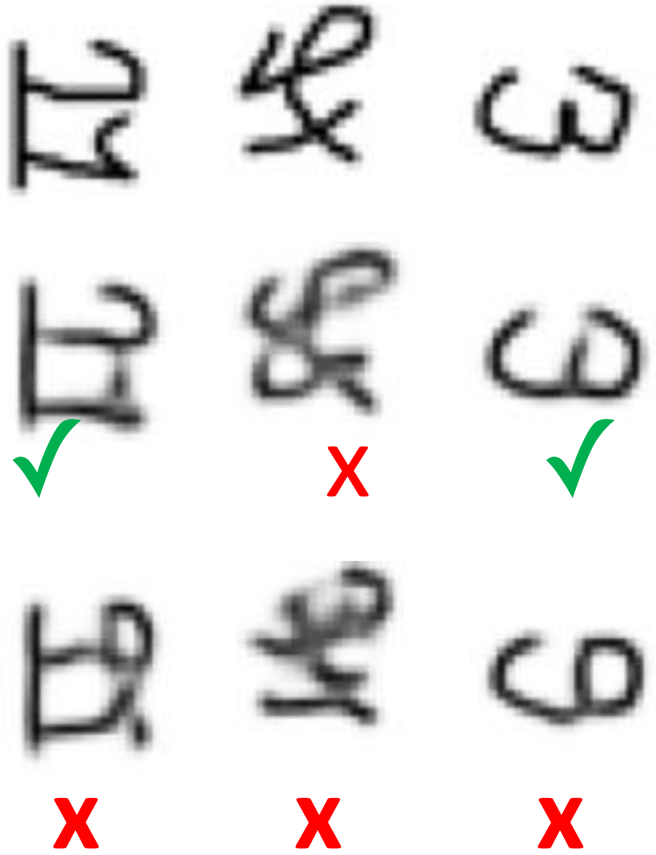
Signal + noise



$$I(Z;X) = -\mathbb{E}\log\left|\det S(\mathbf{x})\right|$$

Better compression reduces distortion at low rates

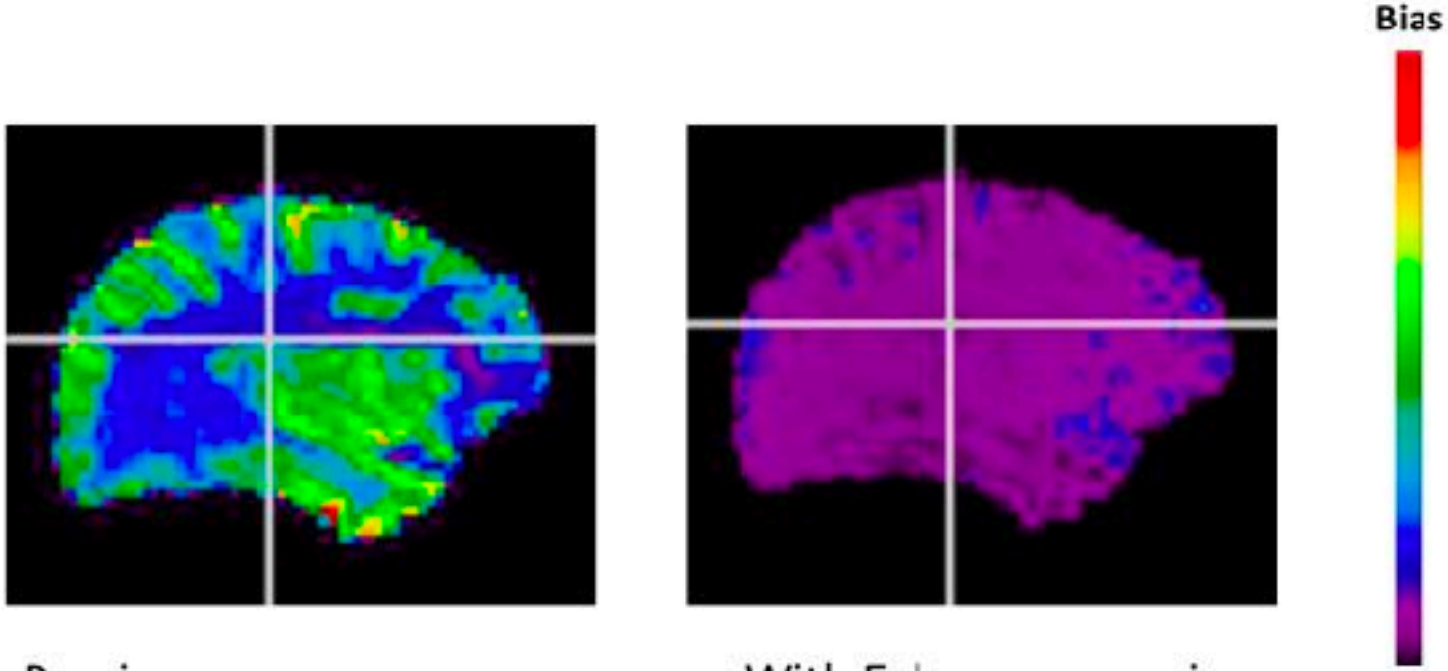
Lossy reconstruction a rate of 30 nats



	<u>**Average Distortion**</u>
Original	
Echo	**83.6 nats**
VAE with inverse autoregressive flow	**85.3 nats**

Invariance = compression (NeurIPS 2018, arXiv:1805.09458), so we can use echo for invariant representation learning

Example: compress away fMRI scanner “site bias”: arXiv:1904.05375



Previous
(Mirzaalian, 2018)

With Echo compression
Moyer et al,
arxiv:1904.05375

More bias → Less bias

Conclusion

- Echo is a more powerful noise model, with an exact, analytic formula for the mutual information (instead of loose bounds)
- Useful wherever lossy compression / rate-distortion appear:
  - (supervised) information bottleneck
  - VAE
  - invariant representation learning
  - Disentangling

Paper: arXiv:1904.07199, NeurIPS 2019

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Code: https://github.com/brekelma/echo