Mathematical Model of Foraging in a Group of Robots: Effect of Interference

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Abstract

In multi-robot applications, such as foraging or collection tasks, interference, which results from competition for space between spatially extended robots, can significantly affect the performance of the group. We present a mathematical model of foraging in a homogeneous multi-robot system, with the goal of understanding quantitatively the effects of interference. We examine two foraging scenarios: a simplified collection task where the robots only collect objects, and a foraging task, where they find objects and deliver them to some pre-specified "home" location. In the first case we find that the overall group performance improves as the system size grows; however, interference causes this improvement to be sublinear, and as a result, each robot's individual performance decreases as the group size increases. We also examine the full foraging task where robots collect objects and deliver them home. We find an optimal group size that maximizes group performance. For larger group sizes, the group performance declines. However, again due to the effects of interference, the individual robot's performance is a monotonically decreasing function of the group size. We validate both models by comparing their predictions to results of sensor-based simulations in a multi-robot system and find good agreement between theory and simulations data.

1 Introduction

Robot collection and foraging are two of the oldest and most studied problems in robotics. In these tasks a single robot or a group of robots [Goldberg and Matarić, 2000, Parker, 1994] has to collect objects scattered around the arena and to assemble them either in some random location (collection task [Beckers *et al.*, 1994, Martinoli *et al.*, 1999]) or a pre-specified "home" location (foraging task [Matarić, 1992, Goldberg and Matarić, 2000, Nitz *et al.*, 1993]). These tasks have been studied under a wide variety of conditions and architectures, both experimentally and in simulation: in homogeneous [Goldberg and Matarić, 2000] and heterogeneous [Goldberg and Matarić, 2000, Parker, 1994] systems, using behavior-based [Matarić, 1992, Goldberg and Matarić, 2000] and hybrid control [Nitz *et al.*, 1993], no communication [Goldberg and Matarić, 2000], direct communication [Nitz *et al.*, 1993, Sugawara and Sano, 1997], as well as indirect communication through the environment [Holland and Melhuish, 2000, Vaughan *et al.*, 2000b]. The broad appeal of this problem is explained both by ubiquity of collection in general and foraging in particular in nature — as seen in the food gathering behavior of many insects — as well as its relevance to many military and industrial applications, such as de-mining, mapping and toxic waste clean-up.

There are several reasons to study foraging in a group of robots. Besides providing a test-bed for the design of physical robots and their controllers, foraging serves as a useful framework for exploring many issues in the design and implementation of multi-robot teams. Additionally, deploying a team of robots to perform a collection or a foraging task is often of practical importance: it introduces robustness and parallelism. Many robots working in parallel may complete the task faster. Additionally, the performance of the group may not be affected by individual robot failure. However, having a task done by a team of robots working together introduces problems not present in single robot systems. One of the most critical issues is the effect of interference, as manifested by collision avoidance, on the performance of the group. Interference has long been recognized as an important issue in multi-robot design. [Fontan and Matarić, 1996, Sugawara and Sano, 1997] While most of the research has concentrated on minimizing interference (through communication, collaboration), few attempts [Nitz *et al.*, 1993] have been made to characterize it and study it quantitatively.

Biological metaphor has been successfully applied to the design of controllers for collection and foraging tasks in reactive [Holland and Melhuish, 2000, Martinoli *et al.*, 1999] and behavior-based [Matarić, 1992, Goldberg and Matarić, 2000] multi-robot systems. In addition to providing a distributed control mechanism, biologically metaphor offers several other advantages for collective robotics systems over alternative designs: (i) scalability: each robot has the same controller whether the group is composed of ten or 10,000 robots; (ii) robustness: group performance is robust to individual agent failure; (iii) flexibility: robots can be dynamically added or removed without significantly affecting the performance of the system; (iv) local sensing: in many cases the desired collective behavior can be achieved via local interactions only; (v) adaptability: allows for simple learning that enables robots to operate in uncertain hostile environments.

A distributed control mechanism based on simple local interactions offers still another advantage over alternative designs: it can be mathematically modeled and analyzed. Mathematical analysis is an alternative to experiment and simulation, two principal tools used by the robotics community. While experiments allow researchers an opportunity to study the systems's behavior in real environments, they are very costly and time-consuming, both to set-up and execute. Sensor-based simulations, such as Webots [Michel, 1998] and Player/Stage [Gerkey *et al.*, 2001], recreate the experiments under realistic conditions, but they usually don't scale well as the size of the system grows, and are, therefore, impractical tools for a detailed investigation of the properties of some multi-robot systems. As a result, many of the questions remain unanswered: *e.g.*, for a given task size, is there an optimal number of robots that will complete the task in the shortest period of time? Using mathematical analysis, on the other hand, we can efficiently study dynamics of even very large robot groups, predict their long term behavior, gain insight into system design: *e.g.*, what parameters determine group behavior, optimize performance, prevent instabilities, *etc.* However, with the exception of the research by Sugawara and coworkers [Sugawara and Sano, 1997, Sugawara *et al.*, 1997], multi-robot systems in general, and foraging in particular, have not been analyzed mathematically.

In this paper, we present a mathematical model of foraging in a homogeneous group of robots. We analyze the behavior of the system, focusing on quantitative characterization of the effects of interference on the performance of the group. In Section 3.1 we introduce and analyze a model of the simplified collection scenario in which the robots find and collect pucks but don't bring them home. In Section 3.2 we examine the full foraging scenario that includes delivering the pucks home. We validate both models by comparing their predictions to results of sensor-based simulations of foraging in a multi-robot system.

2 Foraging in a group of robots

We present a mathematical model of foraging in a homogeneous group of robots using behavior-based control, the type of system studied by Mataric and collaborators [Matarić, 1992, Goldberg and Matarić, 2000]. Figure 1 is a snapshot of a typical experiment with four robots. The robots' task is to collect small pucks scattered randomly around the arena. The arena itself is divided into a search region and a small "home", or goal, region where the collected pucks are deposited. The "boundary" and "buffer" regions are part of the home region and are made necessary by limitations in the robots' sensing capabilities, as described below. Each robot has an identical set of behaviors governed by the same controller. The behaviors that arise in the collection task are [Goldberg and Matarić, 2000]:

Avoiding obstacles, including other robots and boundaries. This behavior is critical to the safety of the robot.

Wandering or searching for pucks: robot moves forward and at random intervals turns left or right through a random arc. If the robot enters the Boundary region, it returns to the search region. This prevents the robot from collecting pucks that have already been delivered.

Detecting a puck.

Grabbing a puck.

Homing : if carrying a puck, move towards the home location.

Creeping : activated by entering Buffer region. The robot will start using the close-range detectors at this point to avoid the boundaries.

Home : robot drops the puck. This activates the exiting behavior.

Exiting : robot exits the home region and resumes search.

Figure 2 shows the sequence of behaviors that the robot engages in during the foraging task. This graph was constructed automatically by analyzing behavior data from foraging experiments [Goldberg and Matarić, 1999]. The presence of

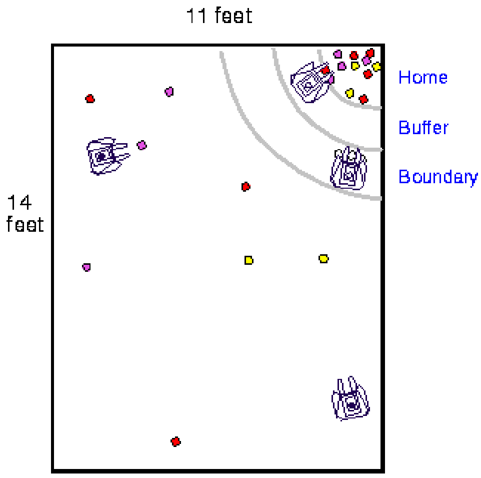

Figure 1: Diagram of the foraging arena (courtesy of D. Goldberg).

three separate avoiding states is necessary to prevent a wandering (searching) robot from making a transition to the homing state through a common avoiding state.

2.1 Interference

In the foraging scenario outlined above, robots act completely independently, without communicating directly or through the environment. Interference is the only interaction between the robots, and it is caused by competition for space between spatially extended robots. When two robots find themselves within sensing distance of one another, they will execute obstacle avoiding maneuvers in order to reduce the risk of a potentially damaging collision. The robot stops, turns in place by some angle and moves forward. This behavior takes time to execute; therefore, avoidance increases the time it takes the robot to find pucks and deliver them home. Clearly, a single robot working alone will not experience interference from other robots. However, if a single robot fails, as is likely in a dynamic, hostile environment, the collection task will not be completed. A group of robots, on the other hand, is robust to an individual's failure. Indeed, many robots may fail but the performance of the group may be only moderately affected. Many robots working in parallel may also speed up the collection task. Of course, the larger the group, the greater the degree of interference — in the extreme case of a crowded arena, robots will spend all their time avoiding other robots and will not bring any pucks home.

Interference has long been recognized as a critical issue in multi-robot systems [Fontan and Matarić, 1996, Sugawara and Sano, 1997]. Several approaches to minimize interference have been explored, including communication [Parker, 1998] and cooperative strategies such as trail formation [Vaughan *et al.*, 2000a] and bucket brigade [Fontan and Matarić, 1996, Østergaard *et al.*, 2001]. In some cases, the effectiveness of the strategy to minimize interference will also depend on the group size [Østergaard *et al.*, 2001]. Therefore, it is important to quantitatively under-

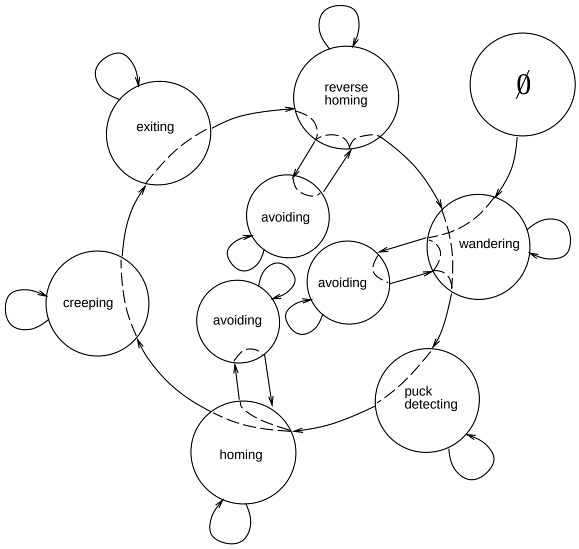

Figure 2: State diagram of the foraging robot behaviors from Goldberg *et al.* [Goldberg and Matarić, 1999]. The diagram was constructed automatically by analyzing behavior data from the foraging experiments.

stand interference between robots and how it relates to the group and task sizes before choosing alternatives to the default strategy. For some tasks and a given controller, there may exist an optimal group size that maximizes the performance of the system [Nitz *et al.*, 1993, Fontan and Matarić, 1996, Østergaard *et al.*, 2001]. Beyond this size the adverse effects of interference become more important than the benefits of increased robustness and parallelism, and it may become beneficial to choose an alternate foraging strategy. We will study interference mathematically and attempt to answer these questions.

2.2 Player/Stage Multi-Robot Simulator

We validate the mathematical model by comparing its predictions to the results of foraging simulations. We used Player/Stage to simulate the foraging task with groups of robots. Player and Stage are a client/server-based scalable multi-robot simulator developed at the USC Robotics Lab [Gerkey *et al.*, 2001]. Player is a network-based interface to the onboard sensors and actuators that constitute a robot, while Stage supports virtual Player robots, sensing and moving in a two-dimensional bitmapped world, that interact with simulated devices. Available sensor models include sonar, laser rangefinder, pan-tilt-zoom camera with color "blob" detection and odometry.

The Stage world consists of a circular arena, with robots and pucks initially randomly distributed around the arena. Each robot comes equipped with a ring of 16 sonars, evenly distributed around its perimeter, for the purpose of obstacle avoidance, a color camera and a vision system to locate "colored" pucks, a gripper

Parameter	Value	Parameter	Value
# of robots	1 - 10	avoid time	3 s
# of pucks	20	avoid dist	250 mm
robot radius	0.2 m	robot speed	300 mm/s
puck radius	0.05 m	min detect area	200 pixels
arena radius	3 m	rev. homing time	10 s
home radius	0.75 m		

Table 1: Simulation parameters

for picking up the puck, and an odometry system to help robot find "home" and move towards it (used in Sec. 3.2 only). We simulated foraging task in groups of one to ten robots, each given a task to collect (or collect and deliver home) 20 pucks. For each group of robots, we averaged results of several, usually ten, simulations. Simulation parameters are listed in Table 1.

Behavior structure The robots' behavior structure closely replicates that of the robots studied in experiments. [Goldberg and Matarić, 2000] Behavior-based control governs the actions of the simulated robots. The following behaviors were used:

0 **Search for pucks**: robot executes a random walk around the arena until a puck is found with a camera. The puck is "painted" some bright color, so that it can be seen with a color camera. The size of the puck in the robot's visual field must exceed some minimum detection area (in pixels), before the robot recognizes it as a puck.

1 **Collect pucks**: under this behavior the robot will visually servo towards the puck and collect it with a gripper. The gripper mail fail to pick up a puck with some small probability, consistent with failure under experimental conditions due to unreliability of real grippers and sensor update rates.

2 **Go home**: after the puck has been collected, the robot will odometrically servo towards the home location and deposit the puck there. Home is a semicircular region centered on a point at the edge of the arena.

3 **Reverse homing**: the robot moves away from home a specified distance in random direction.

4 **Avoid collisions**: If a close obstacle (another robot or arena wall) is sensed at any time, the robot will turn away from the obstacle in a random direction at 40 deg/s for a time specified by the avoid time parameter.

For purposes of analysis only, we split behavior **4** into two distinct behaviors: **4**—avoiding collisions while behaviors **0**, **1** and **3** are active, and **5**—avoiding collisions while homing, *i.e.*, when behavior **2** is active.

3 Mathematical Analysis of Foraging

As mentioned above, interference is the result of competition between two or more robots for the same resource, be it physical space, a puck both are trying to pick up, energy, communications channel, *etc.* In the collection and foraging tasks, competition for physical space, and the resulting avoidance of collisions with other robots, is the most common source of interference. In order to understand interference quantitatively, we will first examine the simplified foraging task that includes searching and avoiding only. This task can be implemented with a subset of robot behaviors listed in Section 1, namely searching, avoiding, detecting a puck and grabbing it. This scenario may be realized experimentally by allowing robots to pick up a puck and store it in a carrying pouch, for instance. Then we will examine the full foraging scenario, where the robots are required to deliver the collected pucks to a home location.

In [Lerman and Galstyan, 2001] we presented a methodology for constructing mathematical models that describe the dynamics of collective behavior of multi-agent systems. The methodology applies to Markov and semi-Markov systems, in which each agent's state at a future time depends only on its present state (and for semi-Markov systems also on how much time the agent has spent in this state) and none of its past states. While this may seem as a restrictive criterion, it is satisfied by many behavior-based and reactive robotics systems. In the context of robotics, *state* labels a set of related robot behaviors required to accomplish a task. Thus, the search state may consist of the *wandering* and *puck detecting* behaviors, or we may simply take each behavior to be a separate state. The mathematical model consists of a series of coupled differential equations, one for each state, each of which describes how the number of agents in that state changes in time. The equations may be solved analytically or numerically, allowing us to quantitatively study the behavior of the multi-agent system. Below we construct and solve a mathematical model of two foraging scenarios, with an emphasis on analyzing the effects of interference.

3.1 Searching and Avoiding

In order to construct a mathematical model of the simplified foraging scenario, the "searching and avoiding"-only case, it is helpful to write down the macroscopic state diagram of the system. During some short time interval, every robot is either in the searching state or the avoiding state, as shown in Fig. 3. We incorporate in the search state actions such as detecting a puck and grabbing it.

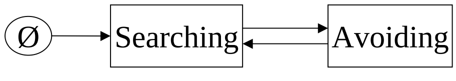

Figure 3: State diagram of the simplified foraging scenario in which robots search and collect pucks, but don't deliver them "home"

All the robots are initially in the searching state. The searching robots wander around the arena, looking for pucks. If a searching robot detects an obstacle, such as another robot, it executes avoiding behavior for a time period τ, after which it resumes the search.[1] If, after avoiding for this time period, the robot still finds its path blocked by the obstacle, it will repeat the avoidance maneuver. This is a good approximation of the experimental realization of the avoiding behavior in which robot avoids by turning in place until it senses free space. If a robot encounters a puck, it picks it up and continues searching. This action does not change the robot's state. Let $N_s(t)$ be the number of robots in the search state at time t, and $N_a(t)$ the number of robots in the avoiding state at time t, with $N_s(t) + N_a(t) = N_0$, the total number of robots, a constant. We model the environment by letting $M(t)$ be the number of uncollected pucks at time t. Also, let α_r be the rate of detecting another robot and α_p the rate of detecting a puck. These parameters connect the model to the experiment, and they are related to the size of the robot and the puck, robot's detection radius and the speed of the robot.

Initially, at $t = 0$, there are N_0 searching robots and M_0 pucks scattered around the arena. The following equations specify the dynamics of the system:

$$\frac{dN_s(t)}{dt} = -\alpha_r N_s(t)[N_s(t) + N_0] + \alpha_r N_s(t-\tau)[N_s(t-\tau) + N_0]\,, \quad (1)$$

$$\frac{dM(t)}{dt} = -\alpha_p N_s(t) M(t)\,. \quad (2)$$

The first equation describes how interference affects searching robots. The meaning of the equation is as follows: the number of searching robots decreases when two searching robots detect each other and commence avoiding maneuvers or when a searching robot detects another robot in the avoiding state; it increases when robots that started avoiding behavior at time $t - \tau$ exit the avoiding state and resume searching. We don't need an equation describing the dynamics of the avoiding robots, because we can compute this quantity using the conservation of the total number of robots N_0. Equation 2 says that the number of uncollected pucks decreases in time because searching robots encounter pucks and pick them up. Note, that the rate at which the pucks are collected is proportional to the number of robots in searching mode, $N_s(t)$.

We rewrite the system of Eqs. 1–2 in dimensionless form using the following variable transformations: $n_s(t) = N_s(t)/N_0$ (fraction of searching robots), $m(t) = M(t)/M_0$ (fraction of uncollected pucks), $\alpha = \alpha_p/\alpha_r$, $t \to \alpha_r N_0 t$, $\tau \to \alpha_r N_0 \tau$(dimensionless time)

$$\frac{dn_s}{dt} = -n_s(t)[n_s(t) + 1] + n_s(t-\tau)[n_s(t-\tau) + 1]\,, \quad (3)$$

$$\frac{dm}{dt} = -\alpha n_s(t) m(t)\,, \quad (4)$$

subject to initial conditions $n_s(0) = 1, m(0) = 1$. Note that of the experimental parameters α_r, α_p and τ, the first equation depends only on τ; therefore, its solutions will also depend only on τ.

Figure 4(a) shows typical solutions of Eqs. 3–4. The fraction of searching robots has a steady state value, which is reached after a period of transient oscillations

[1]The robot will also perceive a wall as an obstacle and try to avoid it. However, we do not include wall avoidance into the model. It contributes a constant term, and its effect decreases as size of the arena grows.

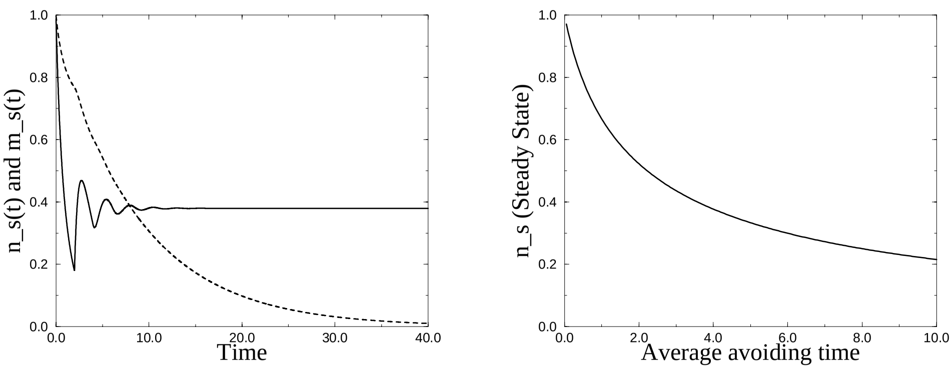

Figure 4: (a) Fraction of searching robots (solid line) and the fraction of uncollected pucks (dashed line) vs. time. (b) Fraction of searching robots in the steady state vs. avoiding time parameter τ.

characteristic of time-delay differential equations. Note, that the steady state value $n_s \equiv n_s(t \to \infty)$ is simply the fraction of time an individual robot spends in the searching mode. Because the avoiding time τ is the only parameter that appears in Eq. 3, the steady state solution is fully determined by τ. Figure 4(b) shows the dependence of the steady state solutions n_s on the avoiding time parameter. Naturally, as the avoiding time increases, a robot spends more time in the avoiding mode, hence decreasing n_s. The dependence of the steady state solution on τ can be obtained using a simple criterion for dynamical equilibrium. Namely, we note that in the steady state the number of robots returning to the searching mode per unit time can be estimated as $n_{av}/\tau \equiv (1 - n_s)/\tau$. Then, balancing the transition rates between two states (searching and avoiding) yields a quadratic equation for n_s with solution

$$n_s = \frac{1}{2}(\sqrt{(1 + 1/\tau)^2 + 4/\tau} - 1 - 1/\tau) \quad (5)$$

Interference depends both on the total number of robots in the system and on the time it takes robots to execute obstacle avoidance behavior. Therefore, we examine next the dependence of the system's performance on the total number of robots. In order to improve performance, is it always beneficial to add robots to the system, or will the negative effects of interference outweigh the added benefits of more workers at some point?

Figure 5(a) shows the number of searching robots in the steady state for two values of the avoiding time parameter as the system size (the total number of robots) grows. The number of searching robots is a monotonically increasing function of the size of the system — no optimal size effect is evident. Moreover, the longer avoiding takes, the fewer searching robots there are at some fixed system size. Figure 5(b) shows the relative efficiency, or the time it takes to collect 80% of pucks per robot, under the same conditions as in (a). We can clearly see that interference adversely affects each robot's *individual* efficiency. The greater the interference effect, either

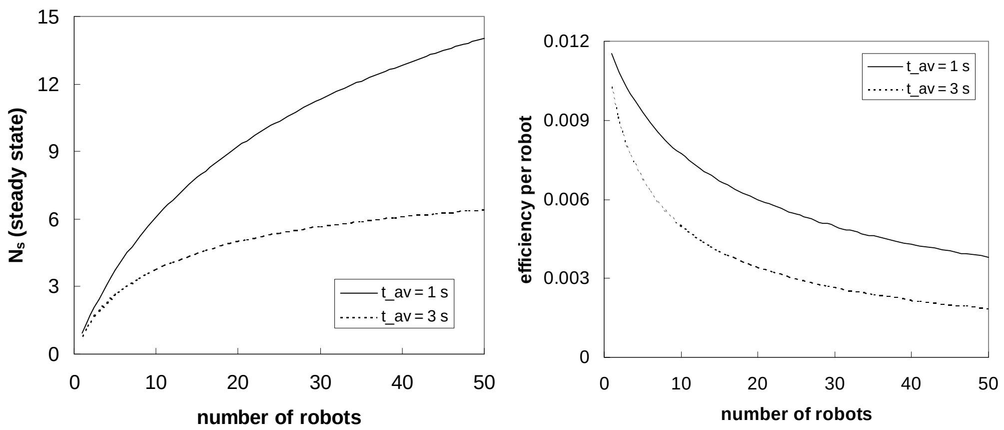

Figure 5: (a) Number of searching robots vs. the total number of robots for two different values of the avoiding time parameter: $\tau = 1s$ (solid line) and $\tau = 3s$ (dashed line). (b) Efficiency per robot vs. system size for the same values of the avoiding time parameter.

because of the larger group size or the bigger avoiding time parameter, the worse the per robot performance is in the foraging task. Thus adding one robot to the system may increase the overall performance of the group, but it will decrease each robot's individual performance.

3.1.1 Comparison with simulations

We ran simulations of the simplified scenario in groups of one to ten robots, where the robots had to find and collect twenty pucks. During these simulations only behaviors **0** (search), **1** (collect), and **4** (avoid) were active (see Section 2.2). Note that the searching state of the mathematical model corresponds to behaviors **0** and **1**, while the avoiding state maps directly to the behavior **4**. We ran the simulation ten times for each group of robots and averaged the results. Each simulation ran until the last of the twenty pucks was collected.

Table 2 lists the average amount of time (in seconds) each robot spent in the active behaviors during the time it took the group to collect all twenty pucks. These values are shown graphically in Fig. 6(a). Although it appears at first that the robots spend less time avoiding other robots as the size of the group grows, because the total time to collect the pucks also decreases, the relative amount of time each robot spends in the avoiding behavior goes up as a function of the group size, as expected (Fig. 6(b)). The last column gives the average number of times a robot attempted to avoid collisions during the same time period. Note the finite number of collisions for a single robot, which are the result of the robot attempting to avoid collisions with walls. Though this number for a single robot is the largest in the column, the rate, or the number of obstacle avoidance maneuvers per unit time, quickly increases as the size of the group grows.

Figure 7(a) shows the time taken by each group of robots to collect all twenty

robots	0	1	4	collisions
1	75.22	71.03	24.15	7.9
2	57.04	37.35	22.19	7.2
3	33.90	24.88	13.61	4.7
4	24.15	18.41	12.75	4.3
5	23.18	14.89	13.50	4.3
6	21.74	13.53	14.93	4.8
7	16.72	11.34	11.24	3.9
8	14.09	9.87	11.06	3.7
9	16.67	9.14	13.31	4.4
10	16.75	7.54	13.96	4.6

Table 2: Average time (in seconds) each robot spends in the active behaviors during the simplified foraging task (**0**: search, **1**: collect, **4**: avoid) during the time it took the group of robots to collect all pucks. The last column gives the average number of obstacle avoidance maneuvers per robot during the same time period.

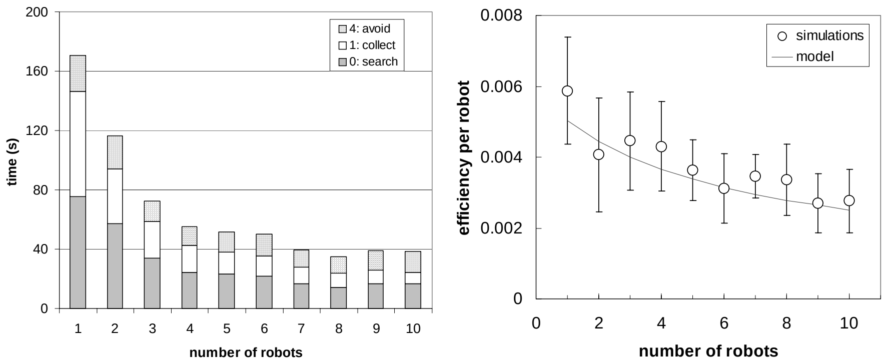

Figure 6: (a) Average time each robot spent in the active behaviors during the time it took the group to collect all pucks vs robot group size. (b) Percentage of time each robot spent in the avoiding behavior as a function of group size.

pucks. The solid line gives the prediction of the mathematical model for $\tau = 3s$, $\alpha_p = 0.02$ and $\alpha_r = 0.04$. The model agrees with the simulations data well within experimental error. Because the time it takes a group of robots to complete the task decreases, the group's efficiency, defined as the inverse of the time it takes the robots to complete the task, increases. However, as pointed out earlier, *efficiency per robot* decreases as the group size grows (*cf.* Fig. 3(b)). The per-robot efficiency, along with the model's prediction, is plotted in Fig. 7(b).

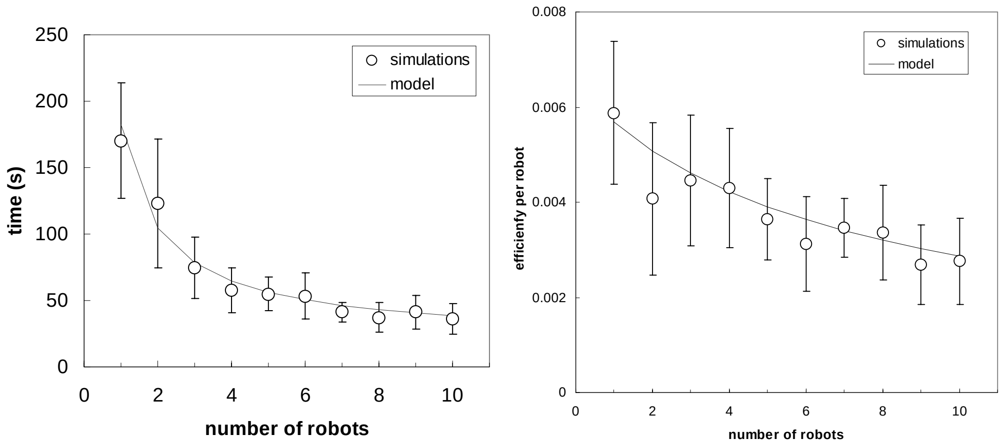

Figure 7: (a) Mean time taken by each group of robots to collect the pucks. The solid line is the prediction of the model with $\tau = 3s$, $\alpha_p = 0.02$ and $\alpha_r = 0.04$. (b) Efficiency per robot.

3.2 Searching, Homing and Avoiding

In the previous section we considered a very simple model of the foraging task consisting of two elementary behaviors, searching for pucks and avoiding obstacles. Our main conclusion was that interference effectively decreases the number of searching robots. We also showed that adding robots always increases overall performance of the system but leads to a deterioration of individual robot performance. In this section we examine the scenario where robots are required to collect pucks and bring them to a specified "home" location.

Figure 8 shows the state diagram for foraging with homing. Initially the robots are in the search state. When a robot finds a puck, it picks it up and moves toward the "home" region. Execution of the homing behavior requires a period of time τ_h. At the end of this period, the robot deposits the puck at home and resumes the search for more pucks. While a robot is either searching or homing, it will encounter and try to avoid other robots for time period τ after which it returns to its previous state. There are two separate avoiding states to preclude robots from moving from the searching to the homing state, or *vice versa*, through the common avoiding state.

The dynamic variables of the new foraging model are $N_s(t), N_h(t), N_{av}^s(t), N_{av}^h(t)$,

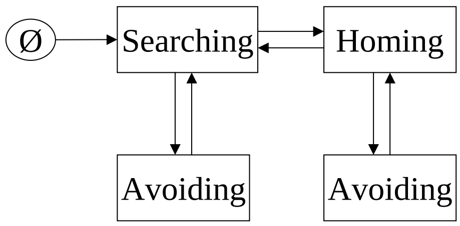

Figure 8: State diagram of a multi-robot foraging system with homing.

the number of searching, homing, avoiding while searching and avoiding while homing robots at time t respectively; and $M(t)$, the number of undelivered pucks in the arena at time t. It was shown experimentally by Goldberg *et al.*[Goldberg and Matarić, 2000] that interference is most pronounced near the home region, because the density of robots will be, on average, greater there. We must, therefore, introduce a new parameter α'_r, the rate of detecting another robot while homing. This number will be different from α_r, because we expect the rate of encountering other robots to be greater near the home region. The following equations describe the time evolution of the dynamic variables:

$$\frac{dN_s(t)}{dt} = -\alpha_p N_s(t)[M(t) - N_h(t) - N^h_{av}(t)] - \alpha_r N_s(t)[N_s(t) + N_0] + \frac{1}{\tau_h} N_h(t) + \frac{1}{\tau} N^s_{av}(t), \quad (6)$$

$$\frac{dN_h(t)}{dt} = \alpha_p N_s(t)[M(t) - N_h(t) - N^h_{av}(t)] - \frac{1}{\tau_h} N_h(t) - \alpha'_r N_h(t)[N_h(t) + N_0] + \frac{1}{\tau} N^h_{av}(t), \quad (7)$$

$$\frac{dN^h_{av}(t)}{dt} = \alpha'_r N_h(t)[N_h(t) + N_0] - \frac{1}{\tau} N^h_{av}(t), \quad (8)$$

$$\frac{dM(t)}{dt} = -\frac{1}{\tau_h} N_h(t), \quad (9)$$

$$N^s_{av}(t) = N_0 - N_s(t) - N_h(t) - N^h_{av}(t). \quad (10)$$

The first two terms in Eq. 6 account for a decrease in the number of searching robots that occurs when robots find and grab pucks, thereby making a transition to the homing state, or when the searching robots encounter and attempt to avoid other robots. The number of available pucks is just the number of pucks in the arena less the pucks held by the homing robots (as well as the homing robots that are temporarily in the avoiding state). The last two terms in the equation require more explanation. We assume that it takes on average τ_h time (in appropriate units) for a robot to reach home after detecting a puck.[2] Then, the average number of robots that deliver the puck during a short time interval dt and return to the searching mode can be approximated as dtN_h/τ_h. Likewise, after a period τ,

[2] In principle, the homing time may depend on the number of robots in the system; however, for the simple model we are considering here we may approximate it by a constant.

dtN^s_{av}/τ robots leave the avoiding state and resume searching. We assume that interference will increase the homing time for each robot: if τ_h^0 is the average homing time in the absence of collisions with other robots, then the effective homing time can be estimated as

$$\tau_h = \tau_h{}^0[1 + \alpha'_r \tau N_0]\,, \qquad (11)$$

where α'_r the probability of encountering a robot while homing.

The remaining equations have similar interpretations. Equation 10 says that we can compute the number of avoiding while searching robots using the fact that the total number of robots is conserved. These equations are approximations of the time delay differential equations we presented earlier (*cf.* Eq. 1). This approximation is valid when looking for long time solutions to the equations, which is exactly the regime we'll be exploring.

Figure 9(a) shows the time evolution of the fraction of searching robots and pucks for $M_0 = 20$, $N_0 = 5$, $\tau = 3s$, $\tau_h^0 = 17s$. The number of searching robots (solid line) first quickly decreases as robots find pucks and carry them home, but then it increases and saturates at some steady state value as the number of undelivered pucks approaches zero (dashed line). The fraction of the searching robots in the steady state depends on the avoiding time parameter, which determines the fraction of robots in the avoiding state.

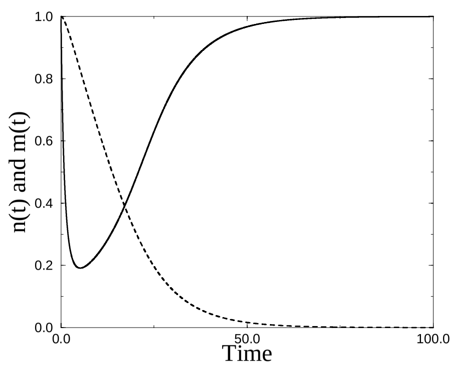

Figure 9: Time evolution of the fraction of searching robots (solid line) and uncollected pucks (dashed line) for $\alpha_p = 0.01$, $\alpha_r = 0.04$, and $\alpha'_r = 0.1$. The number of pucks decreases to zero, during which time the fraction of searching robots approaches one.

In order to compare the performance of different size groups, we define the efficiency of the system as the inverse time required for the group to collect 80% of the pucks ($m(T_{80\%}) = 0.2$ in Fig. 9(b)). Figure 10(a) shows efficiency of the group vs. group size for two different interference strengths, as measured by the avoiding time parameter τ. For both cases the efficiency of the group peaks for some group size and decreases when the number of robots in the group is increased. The efficiency is less for the group with higher interference, or larger avoiding time parameter (dashed line). However, unlike the searching with avoiding, in this case efficiency has a maximum, indicating an optimal group size for the task. Moreover, the greater the effect of interference (larger τ), the smaller the optimal group size.

The final plot (Fig. 10(b)) shows that for this variant of the foraging task interference effects cause the per-robot efficiency to be a monotonically decreasing function of the group size. Thus, adding one new robot to the group decreases the performance of every robot, though if the initial group size was less than the optimal size, adding a robot will increase the overall efficiency of the group.

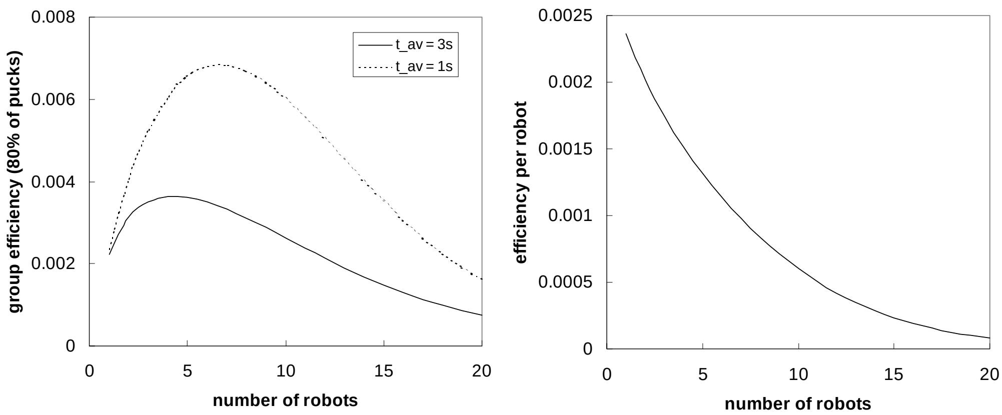

Figure 10: (a) Efficiency of different size robot groups defined as the inverse of the time it takes the group to collect 80% of the pucks in the arena for $\tau = 3s$ (solid line) and $\tau = 1s$ (dashed line). (b) Efficiency per robot for different group sizes.

3.2.1 Comparison with simulations

We ran the simulations of the full foraging scenario in groups of one to ten robots, where the robots had to find and deliver home twenty pucks randomly scattered around the arena. In the results presented below, we split the avoiding behavior into two behaviors: **4**—avoiding while searching, collecting pucks and reverse homing, and **5**—avoiding while the homing behavior is active.

Table 3 lists the average amount of time (in seconds) each robot spent in the active behaviors during the time it took the group collect the pucks and deliver them home. The last two columns list the average number of times a robot attempted to avoid collisions, both while engaging in non-homing behaviors and while homing, during the time it took the group to complete the task. Although in all cases all twenty pucks were collected, robots were only able to deliver on average 19.14 ± 0.533 of them. This was caused by excessive crowding near the home location. In the current implementation of the simulator, robots see the already delivered pucks, and if there are no other pucks left in the arena, the robots will all go home. Although reverse homing acts to disperse robots, and eventually all puck should be delivered, we did not run the simulations long enough for this to happen. The total time in the results presented below is, therefore, the time the last of the pucks was delivered.

Figure 11(a) graphically displays the average amount of time each robot spent

rbts	0	1	2	3	4	5	colls	hcolls
1	307.64	156.90	265.11	225.84	73.69	21.99	23.7	7.1
2	118.68	81.07	170.02	101.89	46.70	45.08	15.1	13.5
3	94.80	61.48	143.22	61.54	57.65	71.78	17.8	22.1
4	50.98	39.71	131.51	34.06	55.84	99.85	15.9	29.5
5	53.14	29.59	126.84	24.27	69.52	150.66	18.9	41.3
6	67.05	28.89	139.68	20.32	94.26	224.40	22.0	53.3
7	137.90	58.11	111.32	23.69	130.21	184.20	37.0	43.1
8	80.94	32.94	133.35	17.06	123.05	265.56	30.1	62.3
9	74.62	31.36	153.58	15.96	130.10	299.18	33.7	77.7

Table 3: Average time (in seconds) each robot spends in the active behaviors during the foraging task (0: search, 1: collect, 2: home, 3: reverse home, 4: avoid, 5: avoid while homing) as a function of robot group size. The last two columns give, respectively, the average number of avoidance maneuvers per robot while searching/collecting/reverse homing and while homing.

in the active behaviors while foraging. Fig. 11(b) shows the fraction of the total time to complete the task that the robot was homing (behaviors **2** and **5**). Note that the rate of increase in the homing time per robot as a function of group size appears to justify our assumption (see Eq. 11) that the homing time increases as the size of the group grows.

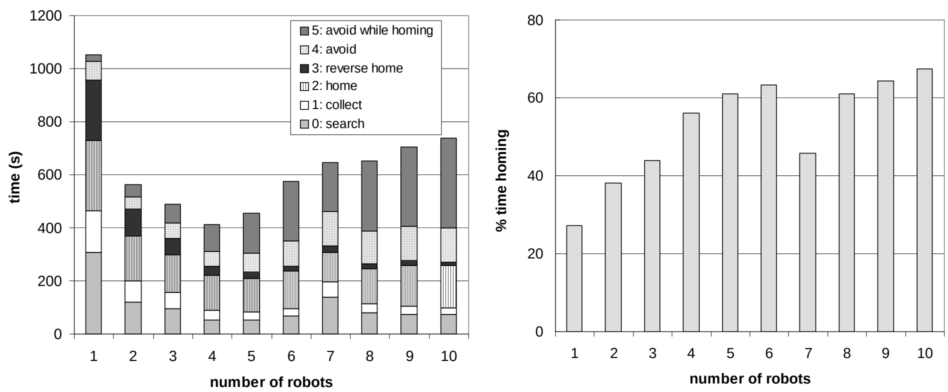

Figure 11: (a) Average time each robot spent in the active behaviors during the time it took the group to deliver all pucks vs robot group size. (b) Percentage of time each robot was homing as a function of group size.

Figure 12(a) shows the total time required to complete the task, *i.e.*, collect all pucks. The solid line is the result of the model's prediction for $\tau^0 = 3s$, $\tau_h^0 = 17s$, $\alpha_p = 0.01$, $\alpha_r = 0.04$, and $\alpha_r' = 0.1$. We also take into account the effect of wall avoidance, it's strength given by $\alpha_w = 0.04$. The simulations data (Fig. 13) shows that the average time per collision increases as the group size grows. This is due to multiple avoidance moves per each attempt to avoid collision, caused by the increase in the local density of robots. Therefore, in the model, we take the avoiding time parameter τ a linear function of N_0, with initial value of $\tau^0 = 3s$. Note that while we chose the parameters that best fit the data, there are differences between theoretical values of some parameters and the simulations data: we took $\tau_h^0 = 17s$, while the data shows the average homing time for a single robot is 14.95 ± 0.98, and $alpha_p = 0.01$, while it should be 0.02. These differences are probably caused by oversimplified approximations, such as a linear dependence of the avoiding and homing time on the group size. Aside from these minor differences in the parameter values, the agreement between model and simulations data is good. Figure 12(b) shows that the per-robot efficiency is a decreasing function of the group size, as predicted by our model.

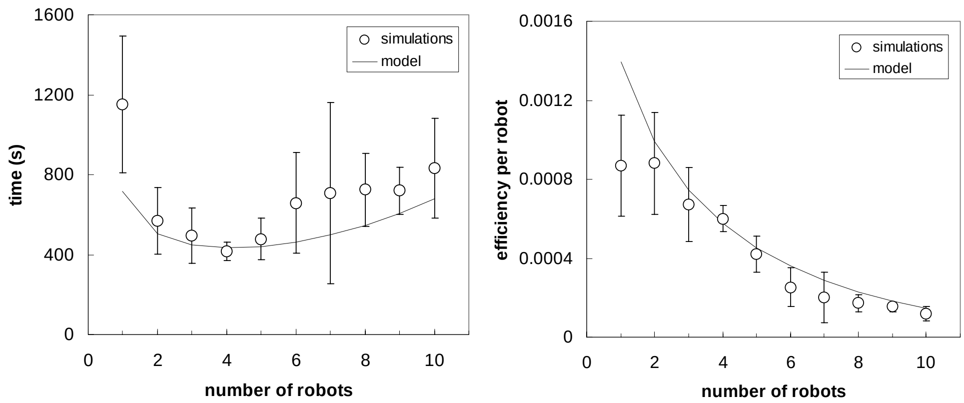

Figure 12: (a) Time it takes the group to collect and deliver pucks home $\tau = 3s$, $\tau_h^0 = 17s$, $\alpha_p = 0.01$, $\alpha_r = 0.04$, $\alpha_r' = 0.1$ and $\alpha_w = 0.04$. (b) Efficiency per robot

4 Related Work

Nitz *et al.*[Nitz *et al.*, 1993] briefly addressed the question of what is an appropriate number of robots for a foraging task in a given environment. By simulating foraging in groups of up to five communicating robots, they observed an increase in performance when adding one to three robots as compared to a single worker. However, the performance seemed to level off and even degrade with further additions. Performance of non-communicating robots seemed to improve as the group size grew, at least up to a group size of five. No simulations for larger group sizes were carried out.

Sugawara and coworkers [Sugawara and Sano, 1997, Sugawara *et al.*, 1998] carried out quantitative studies foraging in groups of communicating and non-communicating robots in different environments. They have developed a simple mathematical model of foraging, similar to ours, and analyzed it under different conditions. In their system when a robot finds a puck, it may broadcast a signal to other robots to move towards it. For non-communicating robots they found that the inverse of the task completion time is linear in the number of robots. This

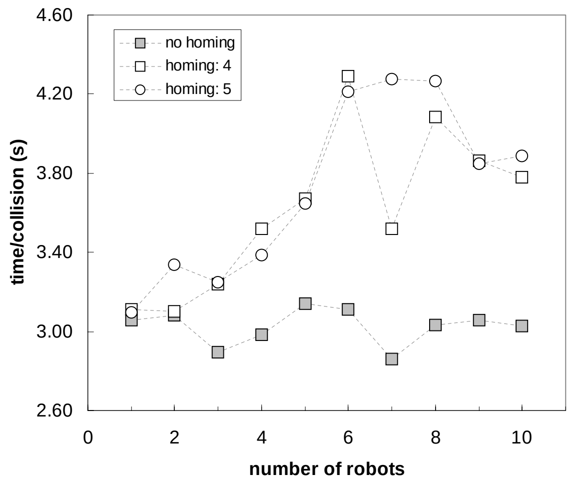

Figure 13: Average time per collision for the search-and-avoid scenario and the foraging scenario, both for the non-homing robots and the homing robots

result was independent of whether the puck distribution was homogeneous or localized. However, in the model describing time evolution of the system, avoiding terms appear only in the equations describing communication between robots. Thus, interference is not taken into account for non-communicating robots. This probably explains the difference between their conclusion and ours. We believe our paper presents a more complete and accurate model of foraging in a homogeneous, albeit non-communicating, multi-robot system.

5 Conclusion

Interference, which results from the inevitable competition for space between spatially extended robots, is an important issue in group robotics. Although we studied it only in the context of foraging by a group of robots, our analysis and, to some extent, conclusions, apply to other multi-robot applications. In order to quantitatively characterize the effects of interference on the group performance, we presented and analyzed a mathematical model of foraging in a group of robots, the type of system studied experimentally by Matarić and collaborators.

We analyzed two foraging scenarios in a homogeneous group of non-communicating robots: the simplified collection task where the robots search for and collect pucks only, and the full foraging task, where they find pucks and deliver them to a pre-specified home location. In the simplified collection task we found that increasing the robot group size reduces the total time required by the group to complete the task, thereby increasing the overall system performance. However, this improvement is sub-linear, and the relative, or the per-robot, foraging efficiency decreases as the size of the group grows. This decrease in the relative performance is due to the effects of interference.

Next, we looked at the system where robots collect pucks and deliver them home.

We included the effects of interference in the homing time, the average time it takes the robots to bring pucks to the home region. Interactions with other robots and the resulting avoiding maneuvers will increase the homing time. In this scenario, there is an optimal group size that maximizes the group's foraging efficiency. System performance decreases for groups larger than optimal size. The value of the optimal group size depends on experimental parameters, and it is smaller the longer it takes the robots to execute obstacle avoidance maneuvers. Although the group efficiency first increases as the size of the group grows, the individual robot efficiency decreases monotonically, again due to the effects of interference.

We ran a number of simulations of both foraging scenarios using Player/Stage, a sensor-based multi-robot simulator, measuring how long it took groups of different sizes to complete the foraging task. We found good agreement between the results of the model and simulations.

The model of foraging presented in this paper is an example of our approach to quantitative mathematical analysis of collective behavior in multi-agent systems. We have applied it to study a number of systems, including coalition formation in electronic marketplaces [Lerman and Shehory, 2000], platoon formation in traffic flow [Galstyan and Lerman, 2001], and collaboration in robots [Lerman *et al.*, 2001]. This class of models describe very simple multi-agent systems in which each agent's future state depends only on its present state and no past states. These models, therefore, cannot take into account memory, learning, and complex decision making abilities. We are extending the mathematical approach to include more complex agents.

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References

[Beckers *et al.*, 1994] R. Beckers, O. E. Holland, and J. L. Deneubourg. From local actions to global tasks: Stigmergy and collective robotics. In Rodney A. Brooks and Pattie Maes, editors, *Proceedings of the 4th International Workshop on the Synthesis and Simulation of Living Systems ArtificialLifeIV*, pages 181–189, Cambridge, MA, USA, July 1994. MIT Press.

[Fontan and Matarić, 1996] Miguel Schneider Fontan and Maja J Matarić. A study of territoriality: The role of critical mass in adaptive task division. In P. Maes, M. J. Matarić, J. A. Meyer, J. Pollack, and S. Wilson, editors, *From Animals to Animats 4: Proceedings of the 4th International Conference on Simulation of Adaptive Behavior*, pages 553–561, Cambridge, MA, 1996. MIT Press.

[Galstyan and Lerman, 2001] A. Galstyan and K. Lerman. A stochastic model of platoon formation in traffic flow. In *Proceedings of the TASK workshop, Santa Fe, NM*, April 17–19 2001.

[Gerkey *et al.*, 2001] Brian P. Gerkey, Richard T. Vaughan, Kasper Sty, Andrew Howard, Gaurav S. Sukhatme, and Maja J Mataric. Most valuable player: A robot device server for distributed control. In *Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2001), Wailea, Hawaii, October 29 - November 3, 2001.* 2001. http://www-robotics.usc.edu/player/.

[Goldberg and Matarić, 1999] Dani Goldberg and Maja J Matarić. Coordinating mobile robot group behavior using a model of interaction dynamics. In *Proceedings of the Autonomous Agents '99*, 1999.

[Goldberg and Matarić, 2000] Dani Goldberg and Maja J Matarić. Robust behavior-based control for distributed multi-robot collection tasks. *submitted to IEEE Transactions on Robotics and Automation.*, 2000.

[Holland and Melhuish, 2000] Owen Holland and Chris Melhuish. Stigmergy, self-organization and sorting in collective robotics. *Artificial Life*, 5(2), 2000.

[Lerman and Galstyan, 2001] K. Lerman and A. Galstyan. A general methodology for mathematical analysis of multi-agent systems. Technical Report ISI-TR529, University of California, Information Sciences Institute, 2001.

[Lerman and Shehory, 2000] Kristina Lerman and Onn Shehory. Coalition Formation for Large-Scale Electronic Markets. In *Proceedings of the International Conference on Multi-Agent Systems (ICMAS'2000), Boston, MA, 2000.*, 2000.

[Lerman *et al.*, 2001] K. Lerman, A. Galstyan, A. Martinoli, and A. Ijspeert. A macroscopic analytical model of collaboration in distributed robotic systems. *submitted to Artificial Life*, 2001.

[Martinoli *et al.*, 1999] A. Martinoli, A. J. Ijspeert, and L. M. Gambardella. A probabilistic model for understanding and comparing collective aggregation mechanisms. In Dario Floreano, Jean-Daniel Nicoud, and Francesco Mondada, editors, *Proceedings of the 5th European Conference on Advances in Artificial Life (ECAL-99)*, volume 1674 of *LNAI*, pages 575–584, Berlin, September 13–17 1999. Springer.

[Matarić, 1992] M. Matarić. Minimizing complexity in controlling a mobile robot population. In *Proceedings of the 1992 IEEE International Conference on Robo tics and Automation*, pages 830–835, Nice, France, 1992.

[Michel, 1998] O. Michel. Webots: Symbiosis between virtual and real mobile robots. In J.-C. Heudin, editor, *Proc. of the First Int. Conf. on Virtual Worlds, Paris, France,*, pages 254–263. Springer Verlag, 1998. See also http://www.cyberbotics.com/webots/.

[Nitz *et al.*, 1993] E. Nitz, R. C. Arkin, and T. Balch. Communication of behavioral state in multi-agent retrieval tasks. In Lisa Werner, Robert; O'Conner, editor, *Proceedings of the 1993 IEEE International Conference on Robotics and Automation: Volume 3*, pages 588–594, Atlanta, GE, May 1993. IEEE Computer Society Press.

[Østergaard *et al.*, 2001] Esben H. Østergaard, Gaurav S. Sukhatme, and Maja J. Matarić. Emergent bucket brigading - a simple mechanism for improving performance in multi-robot constrained-space foraging tasks. In *Proceedings of the 5th International Conference on Autonomous Agents (AGENTS-01)*, 2001.

[Parker, 1994] Lynne E. Parker. Heterogeneous multi-robot cooperation. Technical Report AITR-1465, Massachusetts Institute of Technology, February 1994.

[Parker, 1998] Lynne Parker. Alliance: An architecture for fault-tolerant multirobot cooperation. *IEEE Transactions on Robotics and Automation*, 14(2):220–240, 1998.

[Sugawara and Sano, 1997] Ken Sugawara and Masaki Sano. Cooperative acceleration of task performance: Foraging behavior of interacting multi-robots system. *Physica*, D100:343–354, 1997.

[Sugawara *et al.*, 1997] Ken Sugawara, Masaki Sano, and Ikuo Yoshihara. Cooperative acceleration of task performance: Analysis of foraging behavior by interacting multi-robots. In *Proc. IPSJ Int. Symp. on Information Systems and Technologies for Network Society*, pages 314–317, Fukuoka, Japan, September 1997.

[Sugawara *et al.*, 1998] Ken Sugawara, Masaki Sano, Ikuo Yoshihara, and K. Abe. Cooperative behavior of interacting robots. *Artificial Life and Robotics*, 2:62–67, 1998.

[Vaughan *et al.*, 2000a] Richard T. Vaughan, Kasper Støy, Gaurav S. Sukhatme, and Maja J. Matarić. Blazing a trail: Insect-inspired resource transportation by a robot team. In *Proceedings of the 5th International Symposium on Distributed Autonomous Robotic Systems (DARS), Knoxville, TN*, 2000.

[Vaughan *et al.*, 2000b] Richard T. Vaughan, Kasper Støy, Gaurav S. Sukhatme, and Maja J. Matarić. Whistling in the dark: cooperative trail following in uncertain localization space. In Carlos Sierra, Gini Maria, and Jeffrey S. Rosenschein, editors, *Proceedings of the 4th International Conference on Autonomous Agents (AGENTS-00)*, pages 187–194, NY, June 3–7 2000. ACM Press.