Automatic Loop Parallelization

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Analysis for Parallelism

- Find Data Dependences Across Loop Iterations
- Unlike Data-Flow Analysis Identify Individual Accesses, not aggregate Effects
- Abstraction: For Affine Array Index Functions
  - Linear Inequalities
  - Techniques: Linear Algebra and Integer Programming
Analysis for Parallelism

• Find Data Dependences Across Loop Iterations
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  – Linear Inequalities
  – Techniques: Linear Algebra and Integer Programming
Data Dependence

• **Definition:** Two memory accesses are involved in a data dependence if they may refer to the same memory location and one of the references is a write (not quite complete…).

• **Note:** A Data Dependence can either be between two distinct program statements or two different dynamic executions of the same program statement.

• Two important uses of Data Dependence information:
  – **Parallelization:** if there is not a data dependence between two computations, they may execute safely in parallel.
  – **Locality:** the absence of data dependences eliminates sequential ordering constraints, allowing freedom to reorder for better data locality, also suggests “reuse”
Why is Data Dependence So Important?

• **Basic:** Need to preserve program behavior…

• **Sequential Semantics:** Each Statement Modifies the State of the Execution

• **Goal of Parallelization (reordering):** Reach the same final state - faster!

\[
\begin{align*}
\text{s1: } & \quad a = b + c \\
\text{s2: } & \quad c = 1
\end{align*}
\]
Reordering, Concurrency & Atomicity

- **Reordering:** execution is sequential but order is changed

- **Concurrency:** Statement execution independent at different times

- **Atomicity:** Ensures each statement changes State consistently in a concurrent execution environment, i.e., execution is an interleaving of the execution of the individual statements.

  a: \(a_0\)  \ b: \(b_0\)  \ c: \(c_0\) 

  s1: \(a = b + c\) 

  a: \(b_0 + c_0\)  \ b: \(b_0\)  \ c: \(c_0\) 

  s2: \(c = c+1\) 

  a: \(b_0 + c_0\)  \ b: \(b_0\)  \ c: \(c_0 + 1\)
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\[
\begin{align*}
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\text{s2:} & \quad c = c + 1
\end{align*}
\]

Reordered (still sequential)

\[
\begin{align*}
\text{a: } a_0 & \quad b: b_0 & \quad c: c_0 \\
\text{a: } b_0 & \quad b: b_0 & \quad c: c_0 + 1
\end{align*}
\]
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\[ s1: \text{a} = \text{b} + \text{c} \]
\[ s2: \text{c} = \text{c} + 1 \]

\[ \text{Concurrent (atomic)} \]

\[ a: a_0 \quad b: b_0 \quad c: c_0 \]
\[ a: a_0 \quad b: b_0 \quad c: c_0 \]
\[ a: a_0 \quad b: b_0 \quad c: c_0 \]
\[ a: b_0 + c_0 \quad b: b_0 \quad c: c_0 \]
\[ a: b_0 + c_0 \quad b: b_0 \quad c: c_0 + 1 \]
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```
<table>
<thead>
<tr>
<th>s1: a = b + c</th>
<th>Concurrent (atomic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a: (a_0)</td>
<td>a: (a_0)</td>
</tr>
<tr>
<td>b: (b_0)</td>
<td>b: (b_0)</td>
</tr>
<tr>
<td>c: (c_0)</td>
<td>c: (c_0)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>s2: c = c+1</th>
</tr>
</thead>
<tbody>
<tr>
<td>a: (b_0+c_0)</td>
</tr>
<tr>
<td>b: (b_0)</td>
</tr>
<tr>
<td>c: (c_0+1)</td>
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<tr>
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</tr>
<tr>
<td>c: (c_0+1)</td>
</tr>
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</table>
```

Concurrent (atomic)
Data Dependence for Scalars

- **(A More Comprehensive) Definition:** Two memory accesses are involved in a data dependence if they may refer to the same memory location.

- **Types of Dependence:**
  - True dependence
    \[
    a = \ldots \\
    \ldots = a
    \]
  - Anti-dependence
    \[
    \ldots = a \\
    a = \ldots
    \]
  - Output dependence
    \[
    a = \ldots \\
    a = \ldots
    \]
  - Input Dependence
    \[
    \ldots = a \\
    \ldots = a
    \]

- **In General** for statement \( s_i \) and \( s_j \), a Data dependence exists iff
  - \( s_i \) and \( s_j \) refer to the same variable
  - \( s_i \) executes before \( s_j \)
Parallelization Goal: DOALL Loops

- **DOALL Loops**: Loops whose iterations can execute concurrently (hence in any order)
  - No data dependences
  - Control and Synchronization are trivial

- Example:

  ```
  DO I = 1 TO N
    A(i) = B(i) + C(i)
  ENDDO
  ```

  ```
  DO I = 1 TO N
    spawn task({A(i)=B(i)+C(i)})
  ENDDO
  wait();
  ```
Parallelization Goal: DOALL Loops

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wait();
```
Preliminaries: Loop Normalization

• Normalization allows “base” framework reference for analysis
• Assumes loop iteration counts begin at “1” and step by “1”
• Loops can be normalized to ensure this property:

\[
\begin{align*}
\text{DO I = 4, 12, step 2} & \quad \text{DO I = 1, 5} \\
A(I) = \ldots & \quad A(I*2+2) = \ldots
\end{align*}
\]
Definitions about Reordering

• Definitions:
  – Two computations are equivalent if, on the same inputs,
    • they produce identical outputs
    • the outputs are executed in the same order
  – A reordering transformation changes the order of statement execution without adding or deleting any statement executions.
  – A reordering transformation preserves a dependence if it preserves the relative execution order of the dependences’ source and sink.

• Theorem:
  – Any reordering transformation that preserves every dependence in a program preserves the meaning of that program.
Iteration Space

- $n$-dimensional discrete Cartesian space for $n$ deep loops
- Iteration is represented as coordinates in iteration space
- Sequential execution order of iterations: Lexicographic order

\[ [0,0], [0,1], \ldots, [0,6],[0,7], [1,1], [1,2], \ldots, [1,6], \ldots \]

- Iteration $I$ (a vector) is lexicographically less than $I'$, $I < I'$, iff there exists $c$ \((i_1, \ldots, i_{c-1}) = (i'_1, \ldots, i'_{c-1})\) and $i_c < i'_c$. 

```
DO I = 0, 5
   DO J = I, 7
      ...
```

0 ≤ $i$

\(i \leq 5\)

\(i \leq j\)

\(j \leq 7\)
Distance Vectors

\[
\text{DO } I = 2, N \\
\text{DO } J = 2, N \\
A(I, J) = A(I-1, J-1) + 1
\]

- Distance Vector = [1,1]
- A loop has a Distance Vector (DV) if there exists data dependence from a node \(I\) to a node \(I'\), and \(DV = I' - I\).
- Since \(I' > I\), \(D \geq 0\).
  \(D\) is lexicographically greater than or equal to 0.)
Distance and Direction Vectors

• Distance Vectors: (infinitely large set)

\[
\begin{pmatrix}
0 & 0 & 0 & \ldots & 0 \\
0 & 1 & 2 & \ldots & n
\end{pmatrix}
\begin{pmatrix}
1 & \ldots & 1 \\
-\ldots & 0 & \ldots & n
\end{pmatrix}
\ldots
\begin{pmatrix}
1 & \ldots & 1 \\
-n & \ldots & 0 & \ldots & n
\end{pmatrix}
\]

• Direction Vectors: (realizable if 0 or lexicographically positive)

([=,=],[=,<],[<,>], [<,=], [<,<])

• Common notation:

\begin{align*}
0 &= 0 \\
+ &= < \\
- &= > \\
+/\ &= \ast
\end{align*}
More Distance Vectors Examples

DO I = 2, N
    DO J = 2, N
        A(I,J) = A(I-1,J+1)+1
    END DO
END DO

DO I = 2, N
    DO J = 2, N
        A(I,J) = A(I+1,J-1)+1
    END DO
END DO

DO I = 2, N
    DO J = 2, N
        A(I,J) = A(I,J+1)+1
    END DO
END DO

Question: Which are lexicographically positive?
Parallelization: 1-Dimensional Loop

• Examples:

\[
\begin{align*}
\text{DO } & J = 1, N & \text{DO } & J = 2, N \\
A(J) = & A(J) + 1 & B(J) = & B(J-1) + 1
\end{align*}
\]

• Dependence (Distance and Direction) Vectors:

• Test for parallelization:

  – A loop is parallelizable if for all data dependences \( D \in D \), \( D = 0 \)
Loop-Carried & Loop-Independent Dep.

• A loop-carried dependence occurs between different iteration vectors.

```plaintext
DO I = 1, N
   A(I+1) = A(I) ...
```

• A loop-independent dependence occurs within the same iteration of a loop nest.

```plaintext
DO I = 1, N
   A(I+1) = A(I) ...
```
n-Dimensional Loop Nests

DO I = 1, N
    DO J = 2, N
        A(I, J) = A(I, J-1) + 1
    END DO
END DO

DO I = 2, N
    DO J = 2, N
        A(I, J) = A(I-1, J-1) + 1
    END DO
END DO

- **Definition:**
  $D = (d_1, \ldots, d_n)$ is loop-carried at level $i$ if $d_i$ is the first nonzero element.
Test for Parallelization

The $i$th loop of an $n$-dimensional loop is parallelizable if there does not exist any level $i$ data dependences.

The $i$th loop is parallelizable if for all dependences $D = (d_1, \ldots, d_n)$, either

$$(d_1, \ldots, d_{i-1}) > 0$$

or

$$(d_1, \ldots, d_i) = 0$$
Parallelization Algorithm

• For each pair of array references within the current loop:
  – Determine if there exists a dependence between that pair

• Key points:
  – $n^2$ tests for $n$ accesses in loop!
  – a single access is compared with itself
  – includes accesses in all loops within a nest

• Requires: Good and Quick Dependence Testing Procedure
Dependence Testing

• Question so far:
  – What is the distance/direction (in the iteration space) between two accesses to the same memory location?

• Simpler question:
  – Can two data accesses ever refer to the same memory location?

DO I = 11, 20
A(I) = A(I-1)+ 3

DO I = 11, 20
A(I) = A(I-10)+ 1
Restrict to an Affine Domain

\text{DO } i = 1, N \text{ DO } j = 2i, 100
\begin{align*}
A(i+2j+3, 4i+2j, 3i) &= \ldots \\
\ldots &= A(1, 2i+1, j)
\end{align*}

- Only use loop bounds and array indices which are integer linear functions of loop variables.

- Non-affine examples:
  \text{DO } i= 1, N \text{ DO } j = 1, M
  \begin{align*}
  A(i\times j) &= A(i\times(j-1)) \\
  A(i) &= B(C(i))
  \end{align*}
Equivalence to Integer Programming

• Need to determine if \( F(i) = G(i') \), where \( i \) and \( i' \) are iteration vectors, with constraints \( i, i' \geq L, U \geq i, i' \)

• Example:

\[
\begin{align*}
\text{DO } & I = 2, 100 \\
\text{A}(I) &= \text{A}(I-1)
\end{align*}
\]

• Inequalities:

\[
0 \leq i_1 \leq 100, \quad i_2 = i_1 - 1, \quad i_2 \leq 100
\]

\text{integer vector} \quad I, \quad A I \leq b

• Integer Programming is NP-complete \( \Rightarrow \) Expensive
  
  – \( O(\text{size of the coefficients}) \)
  
  – \( O(n^n) \)
Dependence Testing in the 80s

- Historically, simplify with inexact tests that are more efficient
- Examples: GCD test, Banerjee’s test
- 2 outcomes
  - no dependence
  - maybe a dependence
- Typically, apply a series of more powerful, inexact tests whenever a “maybe” answer is given
- May sacrifice parallelism
Modern Dependence Testing (1991)

• Derive a collection of specific, exact tests that are very efficient

• **Exact tests give two possible answers:** no dependence or definitely a dependence

• Only use inexact tests when exact tests not applicable

• Advantages:
  – exact tests are applicable most of the time
  – avoids cascading of dependence testing when dependence exists

• Example Systems:  SUIF (Stanford), PFC/ParaScope (Rice)
Some Dependence Testing Terms

• **Complexity**: Number of loop indices in a subscript position (ZIV, SIV, MIV)
  
  ```
  DO I
    DO J
      DO K
        A(5, I+1,J) = A(N,I,K) + C
  ```

• **Separability**: whether a given subscript position interacts with other subscripts
  
  ```
  DO I
    DO J
      DO K
        A(I, I,J) = A(I,K,J) + C
  ```
Utility of Separability

• Can independently examine each subscript position
• No precision is lost by simply merging the independent components of a direction vector.
• Subscript positions that are not separable are called coupled.
Example of a Simple, Exact Test

- Strong SIV: An SIV subscript for loop index $I$ is strong if it has the form $<(aI + c_1, aI' + c_2)$

- Dependence distance can be calculated exactly as follows:
  
  \[ d = I' - I = \frac{(c_1 - c_2)}{a} \]
Dependence Testing Overview

- Partition the subscripts into separable and minimal coupled groups.

- Classify each subscript as ZIV, SIV, or MIV.

- For each separable subscript, apply appropriate dependence test. If independence is proved, DONE! Otherwise, produce a set of direction vectors.

- For each coupled group, apply a multiple subscript test and derive direction vectors.

- If any test yields independence, DONE! Otherwise, merge all direction vectors.
Effectiveness of Automatic Parallelization

- Fortran Applications *Automatically* Parallelized by the Stanford SUIF Compiler
- Yielded 50% Higher Specfp95 ratio than previously reported
Summary

- Data dependence is a fundamental concept in compilers for high-performance computing (HPC).
- Data dependence can be used to determine the safety of reordering transformations
  - preserving dependences = preserving “meaning”
- Iteration vectors, distance and direction vectors are abstractions for understanding whether reordering transformations preserve dependences
- Dependence testing has been shown to be equivalent to integer programming
  - can start with simple exact tests
  - can use integer programming techniques
  - can approximate with inexact tests