Data-Flow Analysis

Live-Variable Analysis
Live-Variable Analysis

• What is Live-Variable Analysis?
  – For each Variable $x$ where is the last program point $p$ where the a specific value of $x$ is used.
  – In other words, for $x$ and program point $p$ determine if the value of $x$ at $p$ can still be used along some path starting at $p$.
    • If so, $x$ is live at $p$
    • If not $x$ is dead at $p$
  – Must take Control-Flow into account : a Data-Flow Problem !!!

• Applications:
  – Register Allocation: If a variable is dead at a given point $p$
    • Can reuse its storage, i.e, the register it occupies if any;
    • If its value as been modified must save the value to storage unless it is not live on exit of the procedure or loop
Live-Variable Analysis: Illustration

• At point $p_0$ the $x$ variable is live:
  – There is a path to $p_1$ where value at $p_0$ is used
  – Beyond $p_x$ towards $p_2$ the value of $x$ is no longer needed and is dead

• Need to observe for each variable and for each program point:
  – Where is the last program point beyond which the value is not used
  – Trace back from uses to definitions and observe the first definition (backwards) that reaches that use.
  – That definition kills all uses backwards of it.
Data-Flow Analysis Formulation

- Variable is *live* at a point $p$ if its value is used along *at least one Path*
  - A use of $x$ prior to any definition in basic block means $x$ must be alive
  - A definition of $x$ in $B$ prior to any subsequent use means previous uses must be dead

- Gen Set: Set of Variables Used in $B$
  - Upward Exposed Reads of $B$

- Kill Set: Set of Variables Defined in $B$

\[
\text{OUT}(B) = \bigcup \text{IN}(S) \quad S \text{ a successor of } B
\]

\[
\text{IN}(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B))
\]
Data-Flow Analysis Formulation

• Initialize IN(B) to Empty Set

• Compute Gen/Use and Kill/Def for each Basic Block
  – Tracing backwards from end of block to beginning of block
  – Initialize Last Instruction’s Out(i) to Empty
  – Use IN(i) = use(i) ∪ (OUT(i) - def(i))

• Iteratively Apply Relations to Basic Block Until Convergence
  – OUT(B) = ∪_{S \text{ a successor of } B} IN(s)
  – IN(B) = Use(B) ∪ (OUT(B) - Def(B))

• Given OUT(B) use relations at instruction level to determine the live variables after each instruction
Example

```
i = 0
b = 0
x = p

if(i < p) goto L1

t = a +1
b = t
if (a = b) goto L2

b = b + 1
x = 0

a = x + 1
a = x - 1

i = i + 1
goto L3
```
Use & Def Functions for a Basic Block

\[
\begin{align*}
t &= a + 1 \\
b &= t \\
\text{if } (a = b) \text{ goto } L_2
\end{align*}
\]

\[
\begin{align*}
\text{use } &= \{a\} \\
\text{def } &= \{t\} \\
\text{use } &= \{t\} \\
\text{def } &= \{b\} \\
\text{use } &= \{a, b\} \\
\text{def } &= \{\} \\
\text{In } &= \text{Use } \cup (\text{Out } - \text{Def}) \\
\text{Out } &= \{\} \\
\text{In}(i) &= \text{Use}(i) \cup (\text{Out}(i) - \text{Def}(i))
\end{align*}
\]
Use & Def Functions for a Basic Block

\[
\begin{align*}
\text{t} &= \text{a} + 1 \\
\text{b} &= \text{t} \\
\text{if} \ (\text{a} = \text{b}) \ \text{goto L}_2
\end{align*}
\]

\[
\begin{align*}
\text{use} &= \{a\} \\
\text{def} &= \{t\} \\
\text{use} &= \{t\} \\
\text{def} &= \{b\} \\
\text{use} &= \{a, b\} \\
\text{def} &= \{\} \\
\text{In} &= \{a, b\} \cup (\{\} - \{\}) = \{a, b\} \\
\text{Out} &= \{\} \\
\text{In}(i) &= \text{Use}(i) \cup (\text{Out} (i) - \text{Def}(i))
\end{align*}
\]
Use & Def Functions for a Basic Block

\[
\begin{align*}
  t &= a + 1 \\
  b &= t \\
  \text{if} (a = b) \text{ goto } L_2
\end{align*}
\]

use = \{ a \} \\
def = \{ t \}

use = \{ t \} \\
def = \{ b \}

use = \{ a, b \} \\
def = \{ \}

\text{In} = \text{Use} \cup (\text{Out} - \text{Def})

\text{Out} = \{ a, b \}

\text{Out} = \{ \}

\text{In}(i) = \text{Use}(i) \cup (\text{Out}(i) - \text{Def}(i))
Use & Def Functions for a Basic Block

\[
\begin{align*}
t &= a + 1 \\
b &= t \\
\text{if } (a = b) \text{ goto } L_2
\end{align*}
\]

use = \{ a \} \quad \text{def} = \{ t \}

use = \{ t \} \quad \text{def} = \{ b \}

use = \{ a, b \} \quad \text{def} = \{ \}

In(i) = \text{Use}(i) \cup (\text{Out}(i) - \text{Def}(i))

Out = \{ \}

Out = \{a, b\}

In = \{t\} \cup (\{a, b\} - \{b\})
Use & Def Functions for a Basic Block

\[
\begin{align*}
t &= a + 1 \\
b &= t \\
\text{if } (a = b) \text{ goto } L_2
\end{align*}
\]

\[
\begin{align*}
\text{use} &= \{a\} \\
\text{def} &= \{t\} \\
\text{use} &= \{t\} \\
\text{def} &= \{b\} \\
\text{use} &= \{a, b\} \\
\text{def} &= \{\} \\
\text{Out} &= \{a, t\} \\
\text{Out} &= \{a, b\} \\
\text{Out} &= \{\}
\end{align*}
\]

\[
\text{In}(i) = \text{Use}(i) \cup (\text{Out}(i) - \text{Def}(i))
\]
Use & Def Functions for a Basic Block

In(i) = Use(i) ∪ (Out(i) - Def(i))
Use & Def Functions for a Basic Block

\[
\begin{align*}
\text{In} &= \{a\} \cup (\{a, t\} - \{t\}) \\
\text{Out} &= \{a, t\} \\
\text{Out} &= \{a, b\} \\
\text{Out} &= \{\} \\
\text{In}(i) &= \text{Use}(i) \cup (\text{Out}(i) - \text{Def}(i))
\end{align*}
\]
Use & Def Functions for a Basic Block

\[
\begin{align*}
\text{t} & = a + 1 \\
\text{b} & = t \\
\text{if} \ (a = b) \ \text{goto} \ L_2
\end{align*}
\]

\[
\begin{align*}
\text{use} & = \{ a \} \\
\text{def} & = \{ t \} \\
\text{use} & = \{ t \} \\
\text{def} & = \{ b \} \\
\text{use} & = \{ a, b \} \\
\text{def} & = \{ \} \\
\text{Out} & = \{ a, b \} \\
\text{Out} & = \{ \} \\
\text{In} & = \{ a \} \\
\text{Out} & = \{ a, t \}
\end{align*}
\]

\[
\text{In}(i) = \text{Use}(i) \cup (\text{Out}(i) - \text{Def}(i))
\]
Use & Def Functions for a Basic Block

\[
\text{Use}(B) = \{a\}
\]

\[
t = a + 1 \\
b = t \\
\text{if } (a = b) \text{ goto } L_2
\]

\[
\begin{align*}
\text{use} &= \{a\} \\
\text{def} &= \{t\} \\
\text{use} &= \{t\} \\
\text{def} &= \{b\} \\
\text{use} &= \{a, b\} \\
\text{def} &= \{} \\
\end{align*}
\]

\[
\text{InUse} = \{a\} \\
\text{OutUse} = \{a, t\} \\
\text{OutUse} = \{a, b\} \\
\text{OutUse} = \{}
\]

\[
\text{InUse}(i) = \text{Use}(i) \cup (\text{OutUse}(i) - \text{Def}(i))
\]
Use & Def Functions for a Basic Block

\[
\begin{align*}
\text{t} &= \text{a} + 1 \\
\text{b} &= \text{t} \\
\text{if } (\text{a} = \text{b}) \text{ goto } L_2
\end{align*}
\]

use = \{ a \}
def = \{ t \}

use = \{ t \}
def = \{ b \}

use = \{ a, b \}
def = \{ \}

OutDef = \{ \}\n
InDef(i) = Def(i) \cup OutDef(i)
Use & Def Functions for a Basic Block

\[
t = a + 1 \\
b = t \\
\text{if } (a = b) \text{ goto } L_2
\]

- \( t = a + 1 \):
  - \( \text{use} = \{ a \} \)
  - \( \text{def} = \{ t \} \)

- \( b = t \):
  - \( \text{use} = \{ t \} \)
  - \( \text{def} = \{ b \} \)

- \( \text{if } (a = b) \text{ goto } L_2 \):
  - \( \text{use} = \{ a, b \} \)
  - \( \text{def} = \{ \} \)

\[ \text{InDef} = \text{Def} \cup \text{OutDef} = \{ \} \]
\[ \text{OutDef} = \{ \} \]

\[ \text{InDef}(i) = \text{Def}(i) \cup \text{OutDef}(i) \]
Use & Def Functions for a Basic Block

\[
\begin{align*}
t &= a + 1 & \text{use} &= \{ a \} \quad \text{def} &= \{ t \} \\
b &= t & \text{use} &= \{ t \} \quad \text{def} &= \{ b \} \\
\text{if } (a = b) \text{ goto } L_2 & \quad \text{use} &= \{ a, b \} \quad \text{def} &= \{ \} \\
\end{align*}
\]

InDef = Def \cup OutDef = \{ b \}

OutDef = \{ \}

OutDef = \{ \}

InDef(i) = Def(i) \cup OutDef(i)
Use & Def Functions for a Basic Block

\[ t = a + 1 \]
\[ b = t \]
\[ \text{if } (a = b) \text{ goto L}_2 \]

\( \text{use} = \{ a \} \)
\( \text{def} = \{ t \} \)
\( \text{use} = \{ t \} \)
\( \text{def} = \{ b \} \)
\( \text{use} = \{ a, b \} \)
\( \text{def} = \{ \} \)

\( \text{InDef} = \text{Def} \cup \text{OutDef} = \{ t \} \cup \{ b \} \)
\( \text{OutDef} = \{ b \} \)
\( \text{OutDef} = \{ \} \)
\( \text{OutDef} = \{ \} \)

\( \text{InDef}(i) = \text{Def}(i) \cup \text{OutDef}(i) \)
Use & Def Functions for a Basic Block

\[
\text{Def}(B) = \{t, b\}
\]

\[
t = a + 1 \\
use = \{a\} \\
def = \{t\}
\]

\[
b = t \\
use = \{t\} \\
def = \{b\}
\]

\[
\text{if } (a = b) \text{ goto } L_2 \\
use = \{a, b\} \\
def = \{}
\]

\[
\text{InDef} = \{t, b\}
\]

\[
\text{OutDef} = \{b\}
\]

\[
\text{OutDef} = \{}
\]

\[
\text{OutDef} = \{}
\]

\[
\text{InDef}(i) = \text{Def}(i) \cup \text{OutDef}(i)
\]
Use & Def Functions for a Basic Block

• Can be Accomplished by a Forward Scanning of the Block
  – Keep Track of Which Variables are Read before they are written thus computing the Upwards Exposed Reads (UpExp) or Use Function
  – Track Variables that are Written or Killed (VarKill) or Def Function

```plaintext
// Assume instruction in format “x ← y op z”
for i ← 1 to Num Instructions in B do
  if (instr(i) is leader of B) then
    b ← Number(B);
    UpExp(b) ← ∅;
    VarKill(b) ← ∅;
  if y ∉ VarKill(b) then
    UpExp(b) ← UpExp(b) ∪ {y}
  if z ∉ VarKill(b) then
    UpExp(b) ← UpExp(b) ∪ {z}
  VarKill(b) ← VarKill(b) ∪ {x}
```
Example

```
i = 0
b = 0
x = p

if(i < p) goto L1

b = b + 1
x = 0

t = a + 1
b = t
if (a = b) goto L2

a = x + 1
a = x - 1

i = i + 1
goto L3
```

use = \{ p \}
def = \{ i, b, x \}

use = \{ i, p \}

def = \{ \}

use = \{ b \}
def = \{ x, b \}

use = \{ a \}
def = \{ t, b \}

use = \{ x \}
def = \{ a \}

use = \{ x \}
def = \{ a \}

use = \{ i \}
def = \{ i \}

use = \{ i \}
def = \{ i \}

use = \{ i \}
def = \{ i \}

use = \{ i \}
def = \{ i \}

use = \{ p \}
def = \{ i, b, x \}

use = \{ i, p \}
def = \{ \}

use = \{ b \}
def = \{ x, b \}

use = \{ a \}
def = \{ t, b \}

use = \{ x \}
def = \{ a \}

use = \{ x \}
def = \{ a \}

use = \{ i \}
def = \{ i \}

use = \{ i \}
def = \{ i \}

use = \{ p \}
def = \{ i, b, x \}

use = \{ i, p \}
def = \{ \}

use = \{ b \}
def = \{ x, b \}

use = \{ a \}
def = \{ t, b \}

use = \{ x \}
def = \{ a \}

use = \{ x \}
def = \{ a \}

use = \{ i \}
def = \{ i \}

use = \{ i \}
def = \{ i \}

use = \{ p \}
def = \{ i, b, x \}

use = \{ i, p \}
def = \{ \}

use = \{ b \}
def = \{ x, b \}

use = \{ a \}
def = \{ t, b \}

use = \{ x \}
def = \{ a \}

use = \{ x \}
def = \{ a \}

use = \{ i \}
def = \{ i \}

use = \{ i \}
def = \{ i \}
```
i = 0
b = 0
x = p

if(i < p) goto L

i = i + 1
goto L3
```

```
b = b + 1
x = 0
```

```
t = a + 1
b = t
if (a = b) goto L
```

```
a = x + 1
```

```
a = x - 1
```

```
i = i + 1
```

```
use = {p}
def = {i, b, x}
```

```
use = {i, p}
def = {}
```

```
use = {b}
def = {x, b}
```

```
t = a + 1
b = t
if (a = b) goto L
```

```
a = x + 1
```

```
a = x - 1
```

```
i = i + 1
goto L3
```

```
use = {a}
def = {t, b}
```

```
use = {x}
def = {a}
```

```
use = {i}
def = {i}
```

```
use = {x}
def = {a}
```

```
use = {i}
def = {i}
```

```
use = {x}
def = {a}
```

```
use = {p}
def = {i, b, x}
```

```
use = {i, p}
def = {}
```

```
use = {b}
def = {x, b}
```

```
use = {i, p}
def = {}
```

```
use = {i, p}
def = {}
```
Example

```
i = 0
b = 0
x = p
```

```
if(i < p) goto L_1
```

```
t = a + 1
b = t
if (a = b) goto L_2
```

```
a = x + 1
a = x - 1
```

```
i = i + 1
goto L_3
```

```
b = b + 1
x = 0
```

Use:
- `use = { a }`
- `use = { x }`
- `use = { i }`
- `use = { p }`

Def:
- `def = { i, b, x }`
- `def = { t, b }`
- `def = { x, b }`
- `def = { i }`
- `def = { }`
Example

```
i = 0
b = 0
x = p
```

```
t = a + 1
b = t
if (a = b) goto L2
```

```
ia = x + 1
a = x - 1
```

```
i = i + 1
goto L3
```

OUT(B) = \( \cup \) IN(s)
S a successor of B

IN(B) = Use(B) \( \cup \) (OUT(B) - Def(B))
Example

```
i = 0
b = 0
x = p
```

```
if(i < p) goto L
```

```
t = a +1
b = t
if (a = b) goto L
```

```
a = x + 1
a = x - 1
i = i + 1
goto L
```

\[ \text{IN(B)} = \text{Use(B) } \cup (\text{OUT(B)} - \text{Def(B)}) \]

\[ \text{OUT(B)} = \bigcup \text{IN(s)} \]

\[ S \text{ a successor of B} \]

\[ \text{use} = \{ \text{p} \} \]
\[ \text{def} = \{ \text{i, b, x} \} \]

\[ \text{use} = \{ \text{i, x} \} \]
\[ \text{def} = \{ \text{t, b} \} \]

\[ \text{use} = \{ \text{a} \} \]
\[ \text{def} = \{ \text{t, b} \} \]

\[ \text{use} = \{ \text{x} \} \]
\[ \text{def} = \{ \text{a} \} \]
Example

\[ i = 0 \]
\[ b = 0 \]
\[ x = p \]

\[ t = a + 1 \]
\[ b = t \]
\[ \text{if } (a = b) \text{ goto } L_2 \]

\[ a = x + 1 \]
\[ b = b + 1 \]
\[ x = 0 \]

\[ i = i + 1 \]
\[ \text{goto } L_3 \]

\[ \text{OUT}(B) = \bigcup \text{IN}(s) \]
\[ S \text{ a successor of } B \]

\[ \text{IN}(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B)) \]
Example

\[ \text{OUT}(B) = \bigcup \text{IN}(s) \]
\[ S \text{ a successor of } B \]
\[ \text{IN}(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B)) \]
**Example**

```plaintext
i = 0
b = 0
x = p

Out = \{ i, p, b \}
use = \{ p \}
def = \{ i, b, x \}

if (i < p) goto L_1

Out = \{ b \}
use = \{ b \}
def = \{ x, b \}

In = \{ \}

Out = \{ \}
use = \{ x \}
def = \{ a \}
a = x + 1

Out = \{ i \}
use = \{ i \}
def = \{ a \}
i = i + 1
goto L_3

Out = \{ \}
use = \{ \}
def = \{ \}

Out = \{ i \}
use = \{ i \}
def = \{ i \}
a = x - 1

Out = \{ \}
use = \{ \}
def = \{ \}

Out = \{ \}
use = \{ \}
def = \{ \}

OUT(B) = \bigcup \text{IN}(s) 
\text{S a successor of B}

IN(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B))
```

The diagram illustrates the use and definition of variables in the code snippet, showing the flow of data and dependencies between different sections of the program.
Example

\[
\text{OUT}(B) = \bigcup \text{IN}(s) \\
\text{IN}(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B))
\]

\[
\text{OUT}(B) = \{ i, p, b \} \\
\text{IN}(B) = \{ p \} \\
\text{use} = \{ p \} \\
\text{def} = \{ i, b, x \}
\]

\[
\text{OUT}(B) = \{ b \} \\
\text{IN}(B) = \{ \} \\
\text{use} = \{ i \} \\
\text{def} = \{ \}
\]

\[
\text{OUT}(B) = \{ b \} \\
\text{IN}(B) = \{ b \} \\
\text{use} = \{ b \} \\
\text{def} = \{ x, b \}
\]

\[
\text{OUT}(B) = \{ \} \\
\text{IN}(B) = \{ i \} \\
\text{use} = \{ i \} \\
\text{def} = \{ \}
\]

\[
\text{OUT}(B) = \{ \} \\
\text{IN}(B) = \{ i \} \\
\text{use} = \{ i \} \\
\text{def} = \{ a \}
\]

\[
\text{OUT}(B) = \{ \} \\
\text{IN}(B) = \{ x, i \} \\
\text{use} = \{ x, i \} \\
\text{def} = \{ x, b \}
\]

\[
\text{OUT}(B) = \{ \} \\
\text{IN}(B) = \{ \} \\
\text{use} = \{ i \} \\
\text{def} = \{ a \}
\]

\[
\text{OUT}(B) = \{ \} \\
\text{IN}(B) = \{ i \} \\
\text{use} = \{ i \} \\
\text{def} = \{ a \}
\]
Example

\[ \text{OUT}(B) = \bigcup \text{IN}(S) \]
\[ \text{IN}(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B)) \]

```
i = 0
b = 0
x = p
```

```
t = a + 1
b = t
if (a = b) goto L2
```

```
a = x + 1
b = t
x = 0
```

```
i = i + 1
goto L3
```

```
a = x - 1
```

```
i = i + 1
```

```
i = i
```

```
Out = {} 
In = {} 
```

```
Out = \{i, p, b\} 
In = \{p\} 
```

```
Out = \{i, p\} 
In = \{\} 
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Out = \{i\} 
In = \{\} 
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Out = \{\} 
In = \{\} 
```
Example

\[
\text{OUT}(B) = \bigcup \text{IN}(s) \\
\text{IN}(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B))
\]

\[
\begin{align*}
\text{In} &= \{ p \} \\
i &= 0 \\
b &= 0 \\
x &= p \\
use &= \{ p \} \\
def &= \{ i, b, x \} \\
\text{Out} &= \{ i, p, b \}
\end{align*}
\]

\[
\begin{align*}
\text{In} &= \{ b \} \\
b &= b + 1 \\
x &= 0 \\
use &= \{ b \} \\
def &= \{ x, b \} \\
\text{Out} &= \{} \\
\text{In} &= \{ a, i, x \} \\
t &= a + 1 \\
b &= t \\
if (a = b) \text{ goto } L_2 \\
use &= \{ a \} \\
def &= \{ t, b \} \\
\text{Out} &= \{ b \}
\end{align*}
\]

\[
\begin{align*}
\text{In} &= \{ x, a \} \\
a &= x + 1 \\
use &= \{ x \} \\
def &= \{ a \} \\
\text{Out} &= \{ x, a \} \\
\text{In} &= \{ a \} \\
a &= x - 1 \\
use &= \{ x \} \\
def &= \{ a \} \\
\text{Out} &= \{ a \}
\end{align*}
\]

\[
\begin{align*}
\text{In} &= \{ i \} \\
i &= i + 1 \\
goto L_3 \\
use &= \{ i \} \\
def &= \{ i \} \\
\text{Out} &= \{ a, i, x \} \\
\text{In} &= \{ a, i, x \} \\
\text{Out} &= \{ a, i, x \}
\end{align*}
\]
Example

In = { p }

\[
\begin{align*}
i &= 0 \\
b &= 0 \\
x &= p
\end{align*}
\]

Out = { i, p, b}

use = { p }
def = { i, b, x }

Out(B) = \bigcup \text{IN}(s)

S a successor of B

IN(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B))

Out = { \}

use = { x }
def = { a }

In = { b }

Out = { b }

use = { b }
def = { x, b }

Out = { a, i, x }

In = { a, i, x }

use = { a }
def = { t, b }

Out = { x, a }

if (a = b) goto L2

In = { i, x }

Out = { x, a }

use = { x }
def = { a }

Out = { a, i, x }

Out = { a, i, x }

Out = { a, i, x }

Out = { a, i, x }

Out = { a, i, x }

Out = { a, i, x }

Out = { a, i, x }

Out = { a, i, x }

Out = { a, i, x }

Out = { a, i, x }

Out = { a, i, x }

Out = { a, i, x }

Out = { a, i, x }

OUT(B) = \bigcup \text{IN}(s)

S a successor of B

IN(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B))
Example

\[
\begin{align*}
\text{In} &= \{ p \} \\
i &= 0 \\
b &= 0 \\
x &= p
\end{align*}
\]

\[
\begin{align*}
\text{Out} &= \{ i, p, b \} \\
\text{use} &= \{ p \} \\
\text{def} &= \{ i, b, x \}
\end{align*}
\]

\[
\begin{align*}
\text{In} &= \{ b \} \\
b &= b + 1 \\
x &= 0
\end{align*}
\]

\[
\begin{align*}
\text{Out} &= \{ \} \\
\text{use} &= \{ x \} \\
\text{def} &= \{ a \}
\end{align*}
\]

\[
\begin{align*}
\text{Out} &= \{ a, i, x \} \\
\text{use} &= \{ i, p, b \} \\
\text{def} &= \{ i, b, x \}
\end{align*}
\]

\[
\begin{align*}
\text{Out} &= \{ a, i, x \} \\
\text{use} &= \{ i \} \\
\text{def} &= \{ \}
\end{align*}
\]

\[
\begin{align*}
\text{Out} &= \{ a, x, i \} \\
\text{use} &= \{ x \} \\
\text{def} &= \{ a \}
\end{align*}
\]

\[
\begin{align*}
\text{Out} &= \{ i, p, b \} \\
\text{use} &= \{ \} \\
\text{def} &= \{ \}
\end{align*}
\]

\[
\begin{align*}
\text{Out} &= \{ a, i, x \} \\
\text{use} &= \{ a, i, x \} \\
\text{def} &= \{ a, i, x \}
\end{align*}
\]
Example

\[
\begin{align*}
\text{use} &= \{ \text{p} \} \\
\text{def} &= \{ \text{i, b, x} \}
\end{align*}
\]

\[
\begin{align*}
\text{OUT(B)} &= \bigcup \text{IN(s)} \\
\text{IN(B)} &= \text{Use(B)} \cup (\text{OUT(B)} - \text{Def(B)})
\end{align*}
\]

In = \{ p \}  
\[
\begin{align*}
i &= 0 \\
b &= 0 \\
x &= p
\end{align*}
\]

Out = \{ i, p, b \}

Out = \{ i, p \}  
\[
\begin{align*}
\text{use} &= \{ \text{i, p} \} \\
\text{def} &= \{ \} 
\end{align*}
\]

Out = \{ i, p, b \}  
\[
\begin{align*}
\text{use} &= \{ \text{i, p} \} \\
\text{def} &= \{ \text{i, b, x} \}
\end{align*}
\]

Out = \{ \}  
\[
\begin{align*}
b &= b + 1 \\
x &= 0
\end{align*}
\]

Out = \{ b \}  
\[
\begin{align*}
\text{use} &= \{ \text{b} \} \\
\text{def} &= \{ \text{x, b} \}
\end{align*}
\]

Out = \{ \}  
\[
\begin{align*}
a &= x + 1
\end{align*}
\]

Out = \{ a,i,x \}  
\[
\begin{align*}
\text{use} &= \{ \} \\
\text{def} &= \{ \text{a} \}
\end{align*}
\]

Out = \{ a,i,x \}  
\[
\begin{align*}
a &= x - 1
\end{align*}
\]

Out = \{ a,i,x \}  
\[
\begin{align*}
\text{use} &= \{ \} \\
\text{def} &= \{ \text{a} \}
\end{align*}
\]

Out = \{ i, p, b \}  
\[
\begin{align*}
i &= i + 1 \\
goto \text{L}_3
\end{align*}
\]

Out = \{ a,i,x \}  
\[
\begin{align*}
\text{use} &= \{ \text{i} \} \\
\text{def} &= \{ \text{i} \}
\end{align*}
\]
**Example**

- **In = \{ p \}**
  - `i = 0`
  - `b = 0`
  - `x = p`
  - **Out = \{ i, p, b \}**
    - use = \{ p \}
    - def = \{ i, b, x \}
    - OUT(B) = \bigcup \text{IN}(s)

- **In = \{ b \}**
  - `b = b + 1`
  - `x = 0`
  - **Out = \{ \}**
    - use = \{ b \}
    - def = \{ x, b \}
    - IN(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B))

- **In = \{ a, i, x \}**
  - **Out = \{ i, x \}**
    - `t = a + 1`
    - `b = t`
    - `if (a = b) goto L2`
    - use = \{ a \}
    - def = \{ t, b \}

- **In = \{ i, x \}**
  - **Out = \{ i, x \}**
    - `a = x + 1`
    - use = \{ x \}
    - def = \{ a \}

- **Out = \{ a, i, x \}**
  - **In = \{ a, i, x \}**
    - `a = x - 1`
    - use = \{ x \}
    - def = \{ a \}

- **In = \{ a, i, x \}**
  - **Out = \{ a, i, x \}**
    - `i = i + 1`
    - goto L3
    - use = \{ i \}
    - def = \{ i \}

- **Out = \{ a, i, x \}**

**Definitions**

- OUT(B) = \bigcup \text{IN}(s)
- S a successor of B
- IN(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B))
Example

\[ i = 0 \]
\[ b = 0 \]
\[ x = p \]

\[ t = a + 1 \]
\[ b = t \]
\[ \text{if} (a = b) \quad \text{goto L2} \]

\[ a = x + 1 \]
\[ \text{use} = \{ a \} \]
\[ \text{def} = \{ a \} \]

\[ a = x - 1 \]
\[ \text{use} = \{ x \} \]
\[ \text{def} = \{ a \} \]

\[ i = i + 1 \]
\[ \text{use} = \{ i \} \]
\[ \text{def} = \{ i \} \]

\[ \text{OUT}(B) = \bigcup \text{IN}(S) \]
\[ \text{IN}(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B)) \]
Example

\[ \begin{align*}
\text{In} &= \{ p \} \\
\text{Out} &= \{ a, b, i, p, x \} \\
\text{use} &= \{ p \} \\
\text{def} &= \{ i, b, x \}
\end{align*} \]

\[ \begin{align*}
\text{In} &= \{ b \} \\
\text{Out} &= \{ \} \\
\text{use} &= \{ x \} \\
\text{def} &= \{ a \}
\end{align*} \]

\[ \begin{align*}
\text{Out} &= \{ a, b, i, x \} \\
\text{use} &= \{ b \} \\
\text{def} &= \{ x, b \}
\end{align*} \]

\[ \begin{align*}
\text{In} &= \{ i, x \} \\
\text{Out} &= \{ i, x \} \\
\text{use} &= \{ x \} \\
\text{def} &= \{ a \}
\end{align*} \]

\[ \begin{align*}
\text{In} &= \{ i, x \} \\
\text{Out} &= \{ a, i, x \} \\
\text{use} &= \{ a \} \\
\text{def} &= \{ t, b \}
\end{align*} \]

\[ \begin{align*}
\text{In} &= \{ i, x \} \\
\text{Out} &= \{ i, x \} \\
\text{use} &= \{ x \} \\
\text{def} &= \{ a \}
\end{align*} \]

\[ \begin{align*}
\text{Out} &= \{ a, b, i, x \} \\
\text{use} &= \{ a, i, x \} \\
\text{def} &= \{ a \}
\end{align*} \]

\[ \begin{align*}
\text{Out} &= \{ a, b, i, p, x \} \\
\text{use} &= \{ a, i, x \} \\
\text{def} &= \{ i, b, x \}
\end{align*} \]

\[ \begin{align*}
\text{OUT}(B) &= \bigcup \text{IN}(s) \\
\text{IN}(B) &= \text{Use}(B) \bigcup (\text{OUT}(B) - \text{Def}(B)) \\
S \text{ a successor of } B
\end{align*} \]
Example

\[ i = 0 \]
\[ b = 0 \]
\[ x = p \]

\[ t = a + 1 \]
\[ b = t \]
\[ \text{if } (a = b) \text{ goto } L_2 \]

\[ a = x + 1 \]
\[ a = x - 1 \]

\[ i = i + 1 \]
\[ \text{goto } L_3 \]

\[ i = 0 \]
\[ b = 0 \]
\[ x = p \]

\[ t = a + 1 \]
\[ b = t \]
\[ \text{if } (a = b) \text{ goto } L_2 \]

\[ a = x + 1 \]
\[ a = x - 1 \]

\[ i = i + 1 \]
\[ \text{goto } L_3 \]

\[ \text{OUT}(B) = \bigcup \text{IN}(s) \]
\[ S \text{ a successor of } B \]

\[ \text{IN}(B) = \text{Use}(B) \cup (\text{OUT}(B) - \text{Def}(B)) \]

\[ \text{Out} = \{ a, i, x \} \]

\[ \text{In} = \{ b \} \]

\[ \text{Out} = \{ a, b, i, p, x \} \]

\[ \text{In} = \{ a, i, x \} \]

\[ \text{Out} = \{ a, b, i, x \} \]

\[ \text{In} = \{ i, x \} \]

\[ \text{Out} = \{ i, x \} \]

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\[ \text{In} = \{ a, i, x \} \]

\[ \text{Out} = \{ a, i, x \} \]

\[ \text{Out} = \{ a, i, x \} \]
Example

```
i = 0
b = 0
x = p
```

```
t = a + 1
b = t
if (a = b) goto L2
```

```
Out = { a,b,i,p,x }  
```

```
Out = { a,b,i,x }  
```

```
b = b + 1
x = 0
```

```
Out = {}  
```

```
a = x + 1
```

```
Out = { i,x }  
```

```
t = a + 1
b = t
if (a = b) goto L2
```

```
Out = { i,x }  
```

```
a = x - 1
```

```
Out = { a,i,x }  
```

```
Out = { a,b,i,p,x }  
```

```
Out = { a,b,i,x }  
```

```
Out = { a,i,x }  
```

```
Out = { i,x }  
```

```
Out = { a,i,x }  
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Out = { a,i,x }  
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```
Out = { i,x }  
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```
Out = { a,i,x }  
```

```
Out = { a,i,x }  
```

```
Out = { a,i,x }  
```

```
Out = { a,i,x }  
```

```
Out = { a,i,x }  
```

```
Out = { a,i,x }  
```

```
Out = { a,i,x }  
```

```
Out = {}  
```

```
a = x + 1
```

```
Out = { a,i,x }  
```

```
Out = { a,i,x }  
```

```
i = i + 1
```

```
goto L3
```

```
Out = { a,i,x }  
```
Summary

• What is Live-Variable Analysis?
  – Backward Data-Flow Analysis Problem
  – Upwards Exposed (Gen) - Computed in a Forward Pass

• Most Significant Application
  – Register Allocation