

Randomized Algorithms and NLP: Using Locality Sensitive Hash Functions for High Speed Noun Clustering

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Abstract

In this paper, we explore the power of randomized algorithm to address the challenge of working with very large amounts of data. We apply these algorithms to generate noun similarity lists from 70 million pages. We reduce the running time from quadratic to practically linear in the number of elements to be computed.

1 Introduction

In the last decade, the field of Natural Language Processing (NLP), has seen a surge in the use of corpus motivated techniques. Several NLP systems are modeled based on empirical data and have had varying degrees of success. Of late, however, corpus-based techniques seem to have reached a plateau in performance. Three possible areas for future research investigation to overcoming this plateau include:

1. Working with large amounts of data (Banko and Brill, 2001)
2. Improving semi-supervised and unsupervised algorithms.
3. Using more sophisticated feature functions.

The above listing may not be exhaustive, but it is probably not a bad bet to work in one of the above directions. In this paper, we investigate the first two avenues. Handling terabytes of data requires more efficient algorithms than are currently used in NLP. We propose a web scalable solution to clustering nouns, which employs randomized algorithms. In

doing so, we are going to explore the literature and techniques of randomized algorithms. All clustering algorithms make use of some distance similarity (e.g., cosine similarity) to measure pair wise distance between sets of vectors. Assume that we are given n points to cluster with a maximum of k features. Calculating the full similarity matrix would take time complexity n^2k . With large amounts of data, say n in the order of millions or even billions, having an n^2k algorithm would be very infeasible. To be scalable, we ideally want our algorithm to be proportional to nk .

Fortunately, we can borrow some ideas from the Math and Theoretical Computer Science community to tackle this problem. The crux of our solution lies in defining Locality Sensitive Hash (LSH) functions. LSH functions involve the creation of short signatures (fingerprints) for each vector in space such that those vectors that are closer to each other are more likely to have similar fingerprints. LSH functions are generally based on randomized algorithms and are probabilistic. We present LSH algorithms that can help reduce the time complexity of calculating our distance similarity matrix to nk .

Rabin (1981) proposed the use of hash functions from random irreducible polynomials to create short fingerprint representations for very large strings. These hash function had the nice property that the fingerprint of two identical strings had the same fingerprints, while dissimilar strings had different fingerprints with a very small probability of collision. Broder (1997) first introduced LSH. He proposed the use of Min-wise independent functions to create fingerprints that preserved the Jaccard sim-

ilarity between every pair of vectors. These techniques are used today, for example, to eliminate duplicate web pages. Charikar (2002) proposed the use of random hyperplanes to generate an LSH function that preserves the cosine similarity between every pair of vectors. Interestingly, cosine similarity is widely used in NLP for various applications such as clustering.

In this paper, we perform high speed similarity list creation for nouns collected from a huge web corpus. We linearize this step by using the LSH proposed by Charikar (2002). This reduction in complexity of similarity computation makes it possible to address vastly larger datasets, at the cost, as shown in Section 5, of only little reduction in accuracy. In our experiments, we generate a similarity list for each noun extracted from 70 million page web corpus. Although the NLP community has begun experimenting with the web, we know of no work in published literature that has applied complex language analysis beyond IR and simple surface-level pattern matching.

2 Theory

The core theory behind the implementation of fast cosine similarity calculation can be divided into two parts: **1.** Developing LSH functions to create signatures; **2.** Using fast search algorithm to find nearest neighbors. We describe these two components in greater detail in the next subsections.

2.1 LSH Function Preserving Cosine Similarity

We first begin with the formal definition of cosine similarity.

Definition: Let u and v be two vectors in a k dimensional hyperplane. Cosine similarity is defined as the cosine of the angle between them: $\cos(\theta(u, v))$. We can calculate $\cos(\theta(u, v))$ by the following formula:

$$\cos(\theta(u, v)) = \frac{|u \cdot v|}{\sqrt{|u||v|}} \quad (1)$$

Here $\theta(u, v)$ is the angle between the vectors u and v measured in radians. $|u \cdot v|$ is the scalar (dot) product of u and v , and $|u|$ and $|v|$ represent the length of vectors u and v respectively.

If $\theta(u, v) = 0$, then $\cos(\theta(u, v)) = 1$
and if $\theta(u, v) = \pi/2$, then $\cos(\theta(u, v)) = 0$

The LSH function for cosine similarity as proposed by Charikar (2002) is given by the following theorem:

Theorem: Suppose we are given a collection of vectors in a k dimensional vector space (as written as R^k). Choose a family of hash functions as follows: Generate a spherically symmetric random vector r of unit length from this k dimensional space. We define a hash function, h_r , as:

$$h_r(u) = \begin{cases} 1 & : r \cdot u \geq 0 \\ 0 & : r \cdot u < 0 \end{cases} \quad (2)$$

Then for vectors u and v ,

$$Pr[h_r(u) = h_r(v)] = 1 - \frac{\theta(u, v)}{\pi} \quad (3)$$

Proof of the above theorem is given by Goemans and Williamson (1995). We rewrite the proof here for clarity. The above theorem states that the probability that a random hyperplane separates two vectors is directly proportional to the angle between the two vectors (i.e., $\theta(u, v)$). By symmetry, we have $Pr[h_r(u) \neq h_r(v)] = 2Pr[u \cdot r \geq 0, v \cdot r < 0]$. This corresponds to the intersection of two half spaces, the dihedral angle between which is θ . Thus, we have $Pr[u \cdot r \geq 0, v \cdot r < 0] = \theta(u, v)/2\pi$. Proceeding we have $Pr[h_r(u) \neq h_r(v)] = \theta(u, v)/\pi$ and $Pr[h_r(u) = h_r(v)] = 1 - \theta(u, v)/\pi$. This completes the proof.

Hence from equation 3 we have,

$$\cos(\theta(u, v)) = \cos((1 - Pr[h_r(u) = h_r(v)])\pi) \quad (4)$$

This equation gives us an alternate method for finding cosine similarity. Note that the above equation is probabilistic in nature. Hence, we generate a large (d) number of random vectors to achieve the process. Having calculated $h_r(u)$ with d random vectors for each of the vectors u , we apply equation 4 to find the cosine distance between two vectors. As we generate more number of random vectors, we can estimate the cosine similarity between two vectors more accurately. However, in practice, the number (d) of random vectors required is highly domain dependent, i.e., it depends on the value of the total number of vectors (n), features (k) and the way the vectors are distributed. Using d random vectors, we

can represent each vector by a bit stream of length d .

Carefully looking at equation 4, we can observe that $Pr[h_r(u) = h_r(v)]$ is closely related to the calculation of hamming distance¹ between the vectors u and v , when we have a large number (d) of random vectors r . Thus, the above theorem, converts the problem of finding cosine distance between two vectors to the problem of finding hamming distance between bit streams (as given by equation 4). Finding hamming distance between two bit streams is faster and highly memory efficient. Also worth noting is that this step could be considered as dimensionality reduction wherein we reduce a vector in k dimensions to that of d bits while still preserving the cosine distance between them.

2.2 Fast Search Algorithm

To calculate the fast hamming distance, we use the search algorithm PLEB (Point Location in Equal Balls) first proposed by Indyk and Motwani (1998). This algorithm was further improved by Charikar (2002). This algorithm involves random permutations of the bit streams and their sorting to find the vector with the closest hamming distance. The algorithm given in Charikar (2002) is described to find the nearest neighbor for a given vector. We modify it so that we are able to find the top B closest neighbor for each vector. We omit the math of this algorithm but we sketch its procedural details in the next subsection. Interested readers are further encouraged to read Theorem 2 from Charikar (2002) and Section 3 from Indyk and Motwani (1998).

3 Algorithmic Implementation

In the previous section, we introduced the theory for calculation of fast cosine similarity. We implement it as follows:

1. Initially we are given n vectors in a huge k dimensional space. Our goal is to find all pairs of vectors whose cosine similarity is greater than a particular threshold.
2. Choose d number of ($d \ll k$) unit random vectors $\{r_0, r_1, \dots, r_d\}$ each of k dimensions.

¹Hamming distance is the number of bits which differ between two binary strings. Thus the hamming distance between two strings A and B is $\sum_i |A_i - B_i|$.

A k dimensional unit random vector, in general, is generated by independently sampling a Gaussian function with mean 0 and variance 1, k number of times. Each of the k sample is used to assign one dimension to the random vector. We generate a random number from a Gaussian distribution by using Box-Muller transformation (Box and Muller, 1958).

3. For every vector u , we determine its signature by using the function $h_r(u)$ (as given by equation 4). We can represent the signature of vector u as: $\bar{u} = \{h_{r_1}(u), h_{r_2}(u), \dots, h_{r_d}(u)\}$. Each vector is thus represented by a set of bit streams of length d . Steps 2 and 3 takes $O(nk)$ time (We can assume d to be a constant since $d \ll k$).
4. The previous step gives n vectors, each of them represented by d bits. For calculation of fast hamming distance, we take the original bit index of all vectors and randomly permute them (see Appendix A for more details on random permutation functions). A random permutation can be considered as random jumbling of the bits of each vector (The jumbling is performed by a mapping of the bit index as directed by the random permutation function). A permutation function can be approximated by the following function:

$$\pi(x) = (ax + b) \bmod p \quad (5)$$

where, p is prime and $a, b < p$ AND a and b are chosen at random.

We apply q different random permutation for every vector (by choosing random values for a and b , q number of times). Thus for every vector we have q different bit permutations for the original bit stream.

5. For each permutation function π , we lexicographically sort the list of n vectors (whose bit streams are permuted by the function π) to obtain a sorted list. This step takes $O(n \log n)$ time. (We can assume q to be a constant).
6. For each of the q sorted list (performed after applying the random permutation function π), we

calculate the hamming distance of every vector with B of its closest neighbors in the sorted list. If the hamming distance is below a certain pre-determined threshold, then we output the pair of vectors with their cosine similarity (as calculated by equation 4). Thus, B is the beam parameter of the search. This step takes $O(n)$, since we can assume B, q, d to be a constant.

Why does the fast hamming distance algorithm work? The intuition is that the number of bit streams, d , for each vector is generally smaller than the number of vectors n (ie. $d \ll n$). Thus, sorting the vectors lexicographically after jumbling the bits will likely bring vectors with lower hamming distance closer to each other in the sorted lists.

Overall, the algorithm takes $O(nk + n \log n)$ time. However, for noun clustering, we generally have the number of nouns, n , smaller than the number of features, k . (i.e., $n < k$). This implies $\log n \ll k$ and $n \log n \ll nk$. Hence the time complexity of our algorithm is $O(nk + n \log n) \approx O(nk)$. This is a huge saving from the original $O(n^2k)$ algorithm. In the next section, we proceed to apply this technique for generating noun similarity lists.

4 Building Noun Similarity Lists

A lot of work has been done in the NLP community on clustering words according to their meaning in text (Hindle, 1990; Lin, 1998). The basic intuition is that words that are similar to each other tend to occur in similar contexts, thus linking the semantics of words with their lexical usage in text. One may ask why is clustering of words necessary in the first place? There may be several reasons for clustering, but generally it boils down to one basic reason: if the words that occur rarely in a corpus are found to be distributionally similar to more frequently occurring words, then one may be able to make better inferences on rare words.

However, to unleash the real power of clustering one has to work with large amounts of text. The NLP community has started working on noun clustering on a few gigabytes of newspaper text. But with the rapidly growing amount of raw text available on the web, one could improve clustering performance by carefully harnessing its power. A core component of most clustering algorithms used in the

NLP community is the creation of a similarity matrix. These algorithms are of complexity $O(n^2k)$, where n is the number of unique nouns and k is the feature set length. These algorithms are thus not readily scalable, and limit the size of corpus manageable in practice to a few gigabytes. Clustering algorithms for words generally use the cosine distance for their similarity calculation (Salton and McGill, 1983). Hence instead of using the usual naive cosine distance calculation between every pair of words we can use the algorithm described in Section 3 to make noun clustering web scalable.

To test our algorithm we conduct similarity based experiments on 2 different types of corpus: **1.** Web Corpus (70 million web pages, 138GB), **2.** Newspaper Corpus (6 GB newspaper corpus)

4.1 Web Corpus

We set up a spider to download roughly 70 million web pages from the Internet. Initially, we use the links from Open Directory project² as seed links for our spider. Each webpage is stripped of HTML tags, tokenized, and sentence segmented. Each document is language identified by the software TextCat³ which implements the paper by Cavnar and Trenkle (1994). We retain only English documents. The web contains a lot of duplicate or near-duplicate documents. Eliminating them is critical for obtaining better representation statistics from our collection. One easy example of near duplicate webpage is the presence of different UNIX man pages on the web maintained by different universities. Most of the pages are almost identical copies of each other, except for the presence of email (or name) of the local administrator. The problem of identifying near duplicate documents in linear time is not a trivial. We eliminate duplicate and near duplicate documents by using the algorithm described by Kolcz et al. (2004). This process of duplicate elimination is carried out in linear time and involves the creation of signatures for each document. Signatures are designed so that duplicate and near duplicate documents have the same signature. This algorithm is remarkably fast and has high accuracy.

This entire process of removing non English documents and duplicate (and near-duplicate) docu-

²<http://www.dmoz.org/>

³<http://odur.let.rug.nl/~vannoord/TextCat/>

Table 1: Corpus description

Corpus	Newspaper	Web
Corpus Size	6GB	138GB
Unique Nouns	65,547	655,495
Feature size	940,154	1,306,482

ments reduces our document set from 70 million web pages to roughly 31 million web pages. This represents roughly 138GB of uncompressed text.

We identify all the nouns in the corpus by using a noun phrase identifier. For each noun phrase, we identify the context words surrounding it. Our context window length is restricted to two words to the left and right of each noun. We use the context words as features of the noun vector.

4.2 Newspaper Corpus

We parse a 6 GB newspaper (TREC9 and TREC2002 collection) corpus using the dependency parser Minipar (Lin, 1994). We identify all nouns. For each noun we take the grammatical context of the noun as identified by Minipar⁴. Table 1 gives the characteristics of both corpora. Since we use grammatical context, the feature set is considerably larger than the simple word based proximity feature set.

4.3 Calculating Feature Vectors

Having collected all nouns and their features, we now proceed to construct feature vectors (and values) for nouns from both corpora. We first construct a frequency count vector $C(e) = (c_{e1}, c_{e2}, \dots, c_{ek})$, where k is the total number of features and c_{ef} is the frequency count of feature f occurring in word e . Here, c_{ef} is the number of times word e occurred in context f . We then construct a mutual information vector $MI(e) = (mi_{e1}, mi_{e2}, \dots, mi_{ek})$ for each word e , where mi_{ef} is the pointwise mutual information between word e and feature f , which is defined as:

$$mi_{ef} = \log \frac{\frac{c_{ef}}{N}}{\sum_{i=1}^n \frac{c_{if}}{N} \times \sum_{j=1}^k \frac{c_{ej}}{N}} \quad (6)$$

⁴We perform this operation so that we can compare the performance of our system to that of Pantel and Lin (2002). We do not use grammatical features in the web corpus since parsing is generally not easily web scalable.

where n is the number of words and $N = \sum_{i=1}^n \sum_{j=1}^m c_{ij}$ is the total frequency count of all features of all words. Mutual information is commonly used to measure the association strength between two words (Church and Hanks, 1989).

Having thus obtained the feature representation of each noun we can apply the algorithm described in Section 3 to discover similarity lists. We report results in the next subsection for both corpora.

5 Evaluation

Evaluating clustering systems is generally considered to be quite difficult. However, we are mainly concerned with evaluating the quality and speed of our high speed randomized algorithm. The web corpus is used to show that our framework is web-scalable, while the newspaper corpus is used to compare the output of our system with the similarity lists output by an existing system, which are calculated using the traditional formula as given in equation 1. For this base comparison system we use the one built by Pantel and Lin (2002). We perform 3 kinds of evaluation: **1.** Performance of Locality Sensitive Hash Function; **2.** Performance of fast Hamming distance search algorithm; **3.** Quality of final similarity lists.

5.1 Evaluation of Locality sensitive Hash function

To perform this evaluation, we randomly choose 100 nouns (vectors) from the web collection. For each noun, we calculate the cosine distance using the traditional slow method (as given by equation 1), with all other nouns in the collection. This process creates similarity lists for each of the 100 vectors. These similarity lists are cut off at a threshold of 0.15. These lists are considered to be the gold standard test set for our evaluation.

For the above 100 chosen vectors, we also calculate the cosine similarity using the randomized approach as given by equation 4 and calculate the mean squared error with the gold standard test set using the following formula:

$$error_{av} = \sqrt{\sum_i (CS_{real,i} - CS_{calc,i})^2} \quad (7)$$

Table 2: Error in cosine similarity

Number of random vectors d	Average error in cosine similarity	Time (in hours)
1	1.0000	0.4
10	0.4432	0.5
100	0.1516	3
1000	0.0493	24
3000	0.0273	72
10000	0.0156	241

where $CS_{real,i}$ and $CS_{calc,i}$ are the cosine similarity scores calculated using the traditional (equation 1) and randomized (equation 4) technique respectively. i is the index over all pairs of elements that have $CS_{real,i} \geq 0.15$

We calculate the error ($error_{av}$) for various values of d , the total number of unit random vectors r used in the process. The results are reported in Table 2⁵. As we generate more random vectors, the error rate decreases. For example, generating 10 random vectors gives us a cosine error of 0.4432 (which is a large number since cosine similarity ranges from 0 to 1.) However, generation of more random vectors leads to reduction in error rate as seen by the values for 1000 (0.0493) and 10000 (0.0156). But as we generate more random vectors the time taken by the algorithm also increases. We choose $d = 3000$ random vectors as our optimal (time-accuracy) cut off. It is also very interesting to note that by using only 3000 bits for each of the 655,495 nouns, we are able to measure cosine similarity between every pair of them to within an average error margin of 0.027. This algorithm is also highly memory efficient since we can represent every vector by only a few bits. Also the randomization process makes the the algorithm easily parallelizable since each processor can independently contribute a few bits for every vector.

5.2 Evaluation of Fast Hamming Distance Search Algorithm

We initially obtain a list of bit streams for all the vectors (nouns) from our web corpus using the randomized algorithm described in Section 3 (Steps 1 to 3). The next step involves the calculation of hamming distance. To evaluate the quality of this search algorithm we again randomly choose 100 vectors

⁵The time is calculated for running the algorithm on a single single Pentium IV processor with 4GB of memory

(nouns) from our collection. For each of these 100 vectors we manually calculate the hamming distance with all other vectors in the collection. We only retain those pairs of vectors whose cosine distance (as manually calculated) is above 0.15. This similarity list is used as the gold standard test set for evaluating our fast hamming search.

We then apply the fast hamming distance search algorithm as described in Section 3. In particular, it involves steps 3 to 6 of the algorithm. We evaluate the hamming distance with respect to two criteria: **1.** Number of bit index random permutations functions q ; **2.** Beam search parameter B .

For each vector in the test collection, we take the top N elements from the gold standard similarity list and calculate how many of these elements are actually discovered by the fast hamming distance algorithm. We report the results in Table 3 and Table 4 with beam parameters of ($B = 25$) and ($B = 100$) respectively. For each beam, we experiment with various values for q , the number of random permutation function used. In general, by increasing the value for beam B and number of random permutation q , the accuracy of the search algorithm increases. For example in Table 4 by using a beam $B = 100$ and using 1000 random bit permutations, we are able to discover 72.8% of the elements of the Top 100 list. However, increasing the values of q and B also increases search time. With a beam (B) of 100 and the number of random permutations equal to 100 (i.e., $q = 1000$) it takes 570 hours of processing time on a single Pentium IV machine, whereas with $B = 25$ and $q = 1000$, reduces processing time by more than 50% to 240 hours.

5.3 Quality of Final Similarity Lists

For evaluating the quality of our final similarity lists, we use the system developed by Pantel and Lin (2002) as gold standard on a much smaller data set. We use the same 6GB corpus that was used for training by Pantel and Lin (2002) so that the results are comparable. We randomly choose 100 nouns and calculate the top N elements closest to each noun in the similarity lists using the randomized algorithm described in Section 3. We then compare this output to the one provided by the system of Pantel and Lin (2002). For every noun in the top N list generated by our system we calculate the percentage overlap

Table 3: Hamming search accuracy (Beam $B = 25$)

Random permutations q	Top 1	Top 5	Top 10	Top 25	Top 50	Top 100
25	6.1%	4.9%	4.2%	3.1%	2.4%	1.9%
50	6.1%	5.1%	4.3%	3.2%	2.5%	1.9%
100	11.3%	9.7%	8.2%	6.2%	5.7%	5.1%
500	44.3%	33.5%	30.4%	25.8%	23.0%	20.4%
1000	58.7%	50.6%	48.8%	45.0%	41.0%	37.2%

Table 4: Hamming search accuracy (Beam $B = 100$)

Random permutations q	Top 1	Top 5	Top 10	Top 25	Top 50	Top 100
25	9.2%	9.5%	7.9%	6.4%	5.8%	4.7%
50	15.4%	17.7%	14.6%	12.0%	10.9%	9.0%
100	27.8%	27.2%	23.5%	19.4%	17.9%	16.3%
500	73.1%	67.0%	60.7%	55.2%	53.0%	50.5%
1000	87.6%	84.4%	82.1%	78.9%	75.8%	72.8%

with the gold standard list. Results are reported in Table 5. The results shows that we are able to retrieve roughly 70% of the gold standard similarity list.

In Table 6, we list the top 10 most similar words for some nouns, as examples, from the web corpus.

6 Conclusion

NLP researchers have just begun leveraging the vast amount of knowledge available on the web. By searching IR engines for simple surface patterns, many applications ranging from word sense disambiguation, question answering, and mining semantic resources have already benefited. However, most language analysis tools are too infeasible to run on the scale of the web. A case in point is generating noun similarity lists using co-occurrence statistics, which has quadratic running time on the input size. In this paper, we solve this problem by presenting a randomized algorithm that linearizes this task and limits memory requirements. Experiments show that our method generates cosine similarities between pairs of nouns within a score of 0.03.

In many applications, researchers have shown that more data equals better performance (Banko and Brill, 2001; Curran and Moens, 2002). Moreover, at the web-scale, we are no longer limited to a snapshot in time, which allows broader knowledge to be learned and processed. Randomized algorithms provide the necessary speed and memory requirements to tap into terascale text sources. We hope that ran-

domized algorithms will make other NLP tools feasible at the terascale and we believe that many algorithms will benefit from the vast coverage of our newly created noun similarity list.

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Table 5: Final Quality of Similarity Lists

	Top 1	Top 5	Top 10	Top 25	Top 50	Top 100
Accuracy	70.7%	71.9%	72.2%	71.7%	71.2%	71.1%

Table 6: Sample Top 10 Similarity Lists

JUST DO IT	computer science	TSUNAMI	Louis Vuitton	PILATES
HAVE A NICE DAY	mechanical engineering	tidal wave	PRADA	Tai Chi
FAIR AND BALANCED	electrical engineering	LANDSLIDE	Fendi	Cardio
POWER TO THE PEOPLE	chemical engineering	EARTHQUAKE	Kate Spade	SHIATSU
NEVER AGAIN	Civil Engineering	volcanic eruption	VUITTON	Calisthenics
NO BLOOD FOR OIL	ECONOMICS	HAILSTORM	BURBERRY	Ayurveda
KINGDOM OF HEAVEN	ENGINEERING	Typhoon	GUCCI	Acupressure
If Texas Wasn't	Biology	Mudslide	Chanel	Qigong
BODY OF CHRIST	environmental science	windstorm	Dior	FELDENKRAIS
WE CAN	PHYSICS	HURRICANE	Ferragamo	THERAPEUTIC TOUCH
Weld with your mouse	information science	DISASTER	Ralph Lauren	Reflexology

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Appendix A: Random Permutation Functions

We define $[n] = \{0, 1, 2, \dots, n - 1\}$.

$[n]$ can thus be considered as a set of integers from 0 to $n - 1$.

Let $\pi : [n] \rightarrow [n]$ be a permutation function chosen at random from the set of all such permutation functions.

Consider $\pi : [4] \rightarrow [4]$.

A permutation function π is a one to one mapping from the set of $[4]$ to the set of $[4]$.

Thus, one possible mapping is:

$$\pi : \{0, 1, 2, 3\} \rightarrow \{3, 2, 1, 0\}$$

Here it means: $\pi(0) = 3, \pi(1) = 2, \pi(2) = 1, \pi(3) = 0$

Another possible mapping would be:

$$\pi : \{0, 1, 2, 3\} \rightarrow \{3, 0, 1, 2\}$$

Here it means: $\pi(0) = 3, \pi(1) = 0, \pi(2) = 1, \pi(3) = 2$

Thus for the set $[4]$ there would be $4! = 4 * 3 * 2 = 24$ possibilities. In general, for a set $[n]$ there would be $n!$ unique permutation functions. Choosing a random permutation function amounts to choosing one of $n!$ such functions at random.